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DIGITAL COMPUTER PROGRAMS FOR ROCKET NOZZLE DESIGN AND ANALYSIS

VOLUME V

SINGLE EXPANSION PLUG NOZZLE PERFORMANCE

Prepared under Contract NAS9-2487
for NASA Manned Spacecraft Center

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FOREWORD

This manual provides the necessary background for successful operation of the Single Expansion Plug Nozzle Performance computer program. The manual was prepared under Contract NAS9-2487, Digital Computer Programs for Rocket Nozzle Design and Analysis, with the NASA Manned Spacecraft Center, Houston, Texas, and is the fifth of seven volumes specified in Part I of the contract.

CONTENTS

ABSTRACT

The necessary information for successful operation of the Single Expansion Plug Nozzle Performance computer program is presented in this manual. Boundary conditions for the construction of the supersonic flow field and the order of calculations of the computer program is given with a discussion and flow diagram of each subroutine. The input required by the program is described and a sample output given.

No attempt is made to derive the equations used by the program. A general derivation of the basic equations, along with applications, is given in Volume I of this report.

SECTION I
INTRODUCTION

Since plug nozzles can operate over a wide range of altitudes while sustaining a high level of performance, it is often necessary to calculate the performance for a given plug nozzle at off-design conditions. This can be approximated with the Single Expansion Plug Nozzle Performance computer program, which uses the method of characteristics for steady, supersonic potential flow in developing the flow field.

A general description of the types of nozzle contours, gas models, and starting conditions and the effect of the external boundary is presented herein with a detailed description of the order of calculations used by the computer program. Each of the subroutines used in the program is discussed and flow diagrams are given for clarification. The input and output formats for the program are included with recommended procedures to take in the event of unsuccessful runs.

If strong shock waves are encountered that cannot be approximated by a foldback in the characteristic curves, the program will fail and further calculations at the given off-design condition will be discontinued.

Although a truncated plug nozzle contour may be used, no attempt is made to determine the effects of base pressure on performance.

SECTION II
TECHNICAL DESCRIPTION

The Single Expansion Plug Nozzle Performance computer program uses the method of characteristics for two-dimensional or axisymmetric potential flow to calculate the supersonic flow field and performance for a given plug nozzle contour, gas model, starting condition, and altitude. The nozzle is assumed to exhaust into quiescent air.

The main program of the deck is used to control the order of calculations, while calculations such as determining the fluid properties at an interior point of the flow field or performance parameters along the contour of the nozzle are made in subroutines, which are "called" by the main program or other subroutines. The function of the main program and calculations made therein are described in Paragraphs A and B, and the descriptions of the individual subroutines are given in Paragraph C.

A. BOUNDARY CONDITIONS

The first function of the main program is to call the INPUT Subroutine, which initializes parameters and reads in the input data. The boundary conditions required in the input are described in the following paragraphs.

1. Nozzle Contour

A typical supersonic plug nozzle contour and characteristic net are shown in figure 1.

Two types of nozzle contours may be used by the program.

1. A contour described by the coordinates of points on the curve C_1C_2 of figure 1.
2. A conical contour for a given half angle and nozzle length.

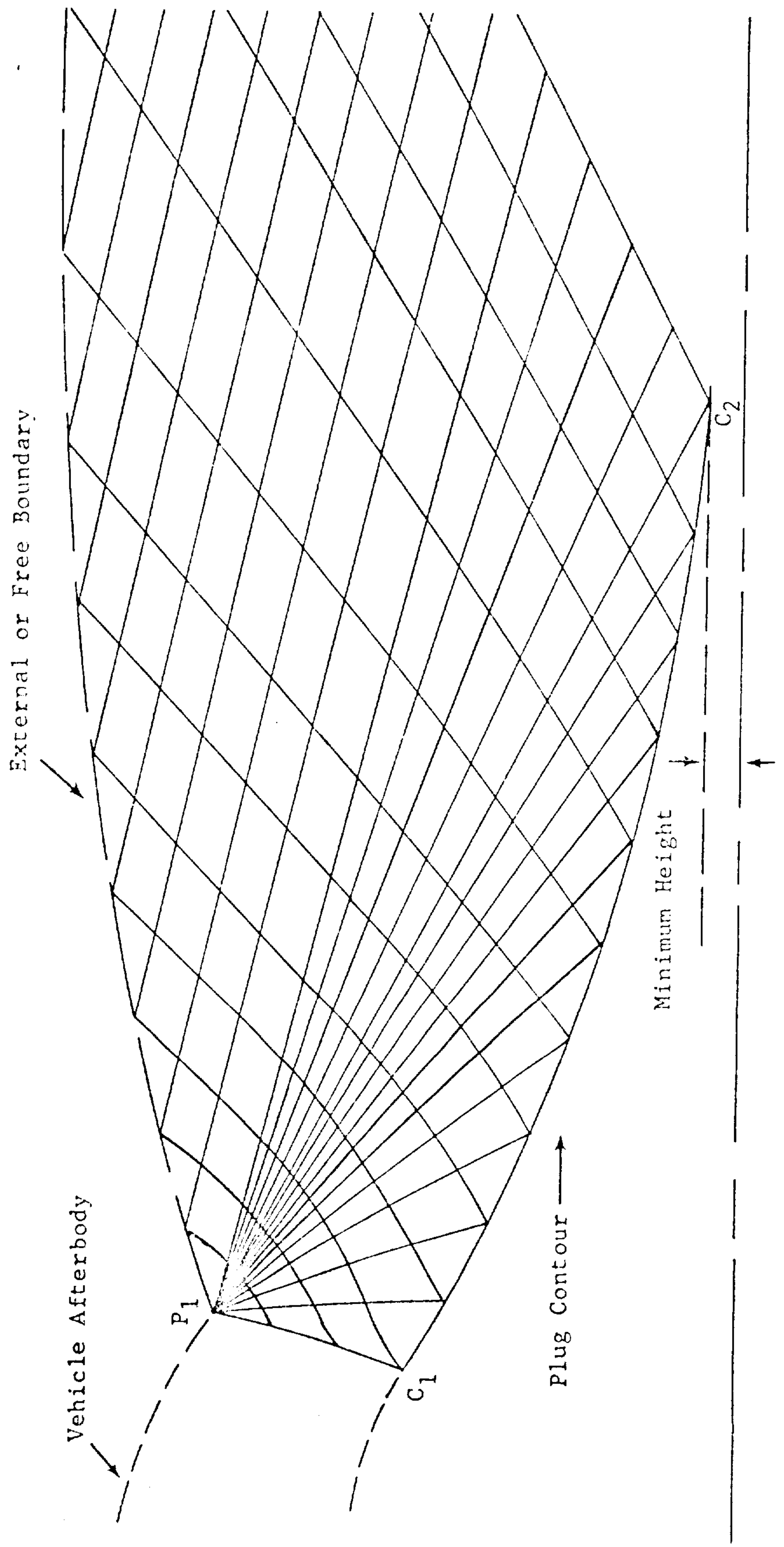

Figure 1. Typical Supersonic Plug Nozzle Contour and Characteristic Net

A semiconical contour may be used by describing the coordinates along the initial part of the contour and providing a nozzle axial length, XCUT, greater than the last input coordinate, but less than the coordinate of the intersection of the cone and the nozzle centerline. A conical section will then be generated from the last coordinate with the slope of this point.

When coordinate points are specified along the contour, a beam fit procedure (BMFIT Subroutine) is used to fit a curve through these points. This method of curve fitting requires that a boundary condition at each end point be specified. In the case of the contour, the boundary conditions are given by the slopes of the contour at the first and last point.

2. Gas Model

In many cases, nozzle performance may be approximated by assuming a perfect gas; in which case only the specific heat ratio, γ, must be input, and the thermodynamic properties are calculated by using perfect gas relationships. For this condition, the local velocity is nondimensionalized with respect to the maximum velocity (V_{max}). The critical velocity ratio, W_{sonic}, (i.e., where M = 1) and density ratio at the throat are

$$W_{sonic} = \frac{V_{sonic}}{V_{max}} = \sqrt{\frac{\gamma - 1}{\gamma + 1}} \; ,$$

and

$$\frac{\rho_{sonic}}{\rho_o} = \left[1 - W^2_{sonic}\right]^{\frac{1}{\gamma-1}} .$$

Nozzle performance for an ideal gas (usually equilibrium or frozen flow) may be calculated by specifying the thermodynamic properties in tabular form as a function of specific impulse. These properties may be obtained from conventional one-dimensional combustion programs*, and consist of pressure, density, and local frozen sound speed (optional), as a function of specific impulse. The SONICP Subroutine beam fits these properties as a function of specific impulse and determines the sonic velocity and density at the throat. The local velocities are then non-dimensionalized with respect to the sonic velocity, and the thermodynamic properties beam fit as a function of the velocity ratio.

3. Starting Conditions

The starting conditions (i.e., the flow properties along P_1C_1 in figure 1) can be specified by any of the following methods:

1. A given Mach line whose coordinates and flow properties are known along the line
2. A straight down Mach line (TMLINE Subroutine)
3. A line of constant Mach number (TMLINE Subroutine).

For all of these cases, the initial throat line must originate at point P_1 in figure 1, and be slightly supersonic.

4. External Boundary

The amount of expansion and the shape of the external or free boundary are dependent upon the altitude at which the performance of the nozzle is to be calculated. Although altitude also affects the pressure on the base of truncated nozzles, no attempt is made to calculate the additional performance due to the base pressure.

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*Zelenik, F. S., and S. Gordon; NASA TN D-1454 "A General IBM 704 or 7090 Computer Program for Computation of Chemical Equilibrium Compositions, Rocket Performance and Chapman - Jouquet Detonations".

When exhausting into quiescent air, the pressure remains constant along the free boundary; therefore, the altitude can be described by the ambient pressure with the exhaust velocity along the free boundary calculated for this pressure (AMBV Subroutine).

B. FLOW FIELD CONSTRUCTION

Having specified the nozzle contour, gas model, starting condition, and ambient pressure, the main program proceeds with the flow field construction and calculates nozzle performance. To calculate performance, the mass flow rate and minimum cross-sectional area are determined by integrating $\rho V_n dA$ along the first down Mach line at the throat (TMFLOW Subroutine). Then the gross thrust coefficient at the first point on the contour is determined by integrating the rate of change of momentum and static pressure forces along the same down Mach line (CTGT Subroutine). Additional thrust is due to the pressure distribution along the plug contour, and is calculated for each flow field contour point. The PERFO Subroutine is used to calculate and print the following performance parameters:

1. X/R
2. Y/R
3. TAN THETA (Contour Slope)
4. MACH NO.
5. P/PC (Local static pressure/chamber pressure)
6. GAMMA (Specific heat ratio)
7. AS/A* (Surface area/throat area)
8. CTG (Gross thrust coefficient)
9. CTN (Net thrust coefficient)

The flow field construction depends on whether the nozzle is over-expanded or underexpanded, as illustrated in figures 2a and 2b. Using the procedure in EXPAND Subroutine, the flow at the throat is expanded through small increments in velocity ratio. From the expansion point,

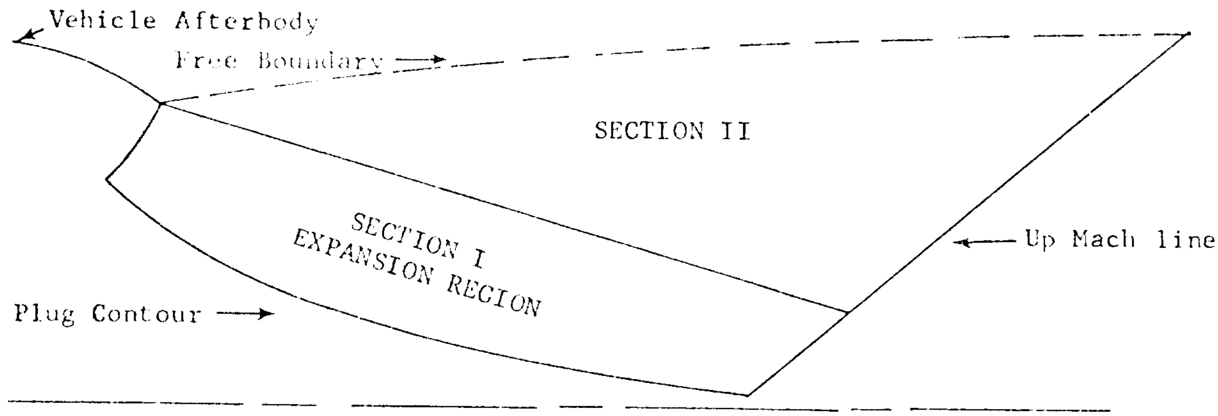


Figure 2a. Underexpanded Nozzle

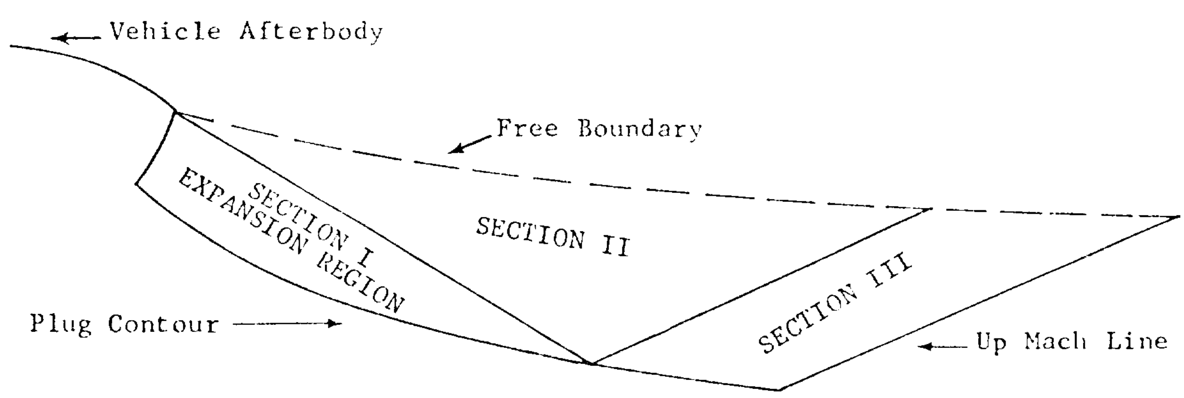


Figure 2b. Overexpanded Nozzle

a down Mach line B_1B_n (figure 3) is constructed by obtaining the intersection of a down Mach line from B_1 with up Mach lines from the points on the initial Mach line A_1A_n (INTX Subroutine). If the end of the contour has not previously been reached, the down Mach line B_1B_n is extended to the nozzle contour using the procedure in BOUNDD Subroutine.

Otherwise, the down Mach lines in the expansion section are terminated at the up Mach line from the last point on the contour.

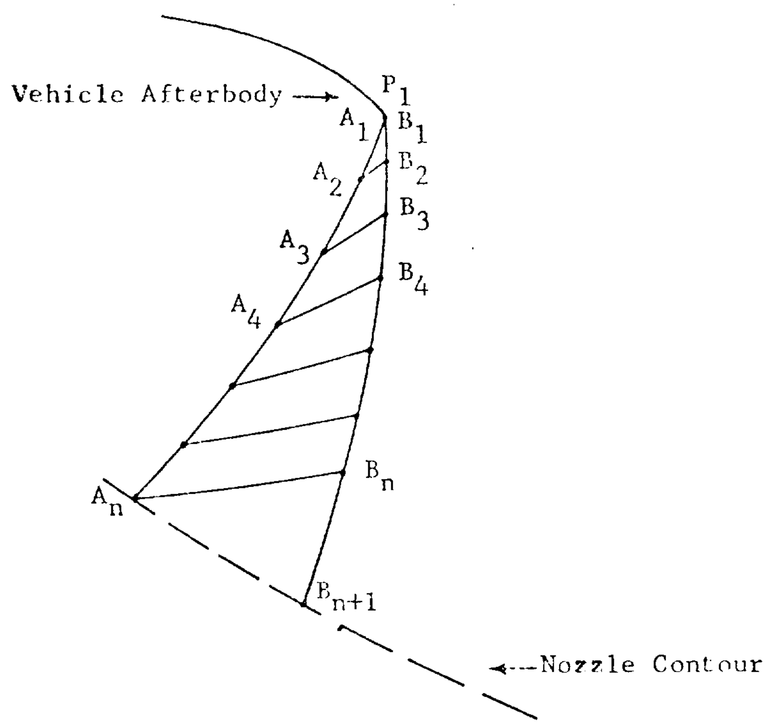


Figure 3

After each boundary calculation, a test is made to check the axial distance between the last two contour intersections. If this distance is greater than a specified tolerance, the expansion increment is halved and the down Mach line reconstructed. The procedure of expanding and constructing new down Mach lines is continued until the expansion is complete.

Section II (figures 2a and 2b) consists of up Mach lines, from the points on the last expansion line, to the free boundary, as shown in figure 4. If the X-coordinate at any point along the up Mach line exceeds the input value of FBCUT, calculations are discontinued on that line and the program proceeds to the next up Mach line.

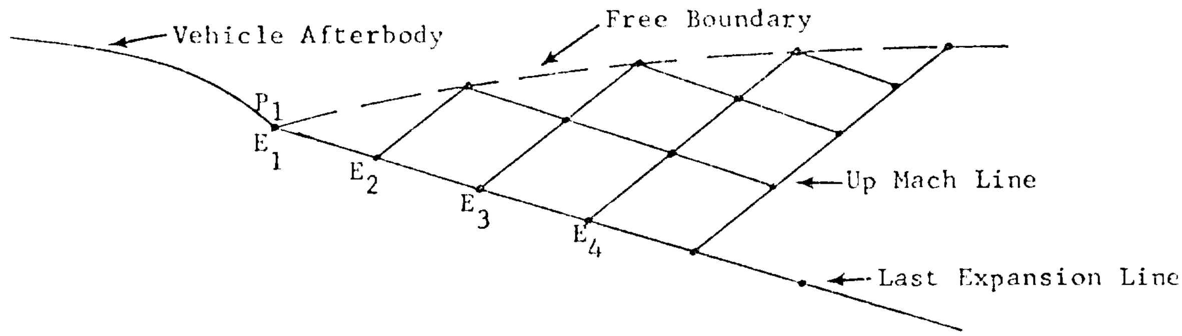


Figure 4

Section III (figure 2b) for an overexpanded nozzle is also constructed with up Mach lines to the free boundary, where the first point on the up Mach line is determined by obtaining the intersection of a down Mach line from the previous up Mach line with the plug contour (figure 5). This procedure is continued until the last point on the contour has been reached.

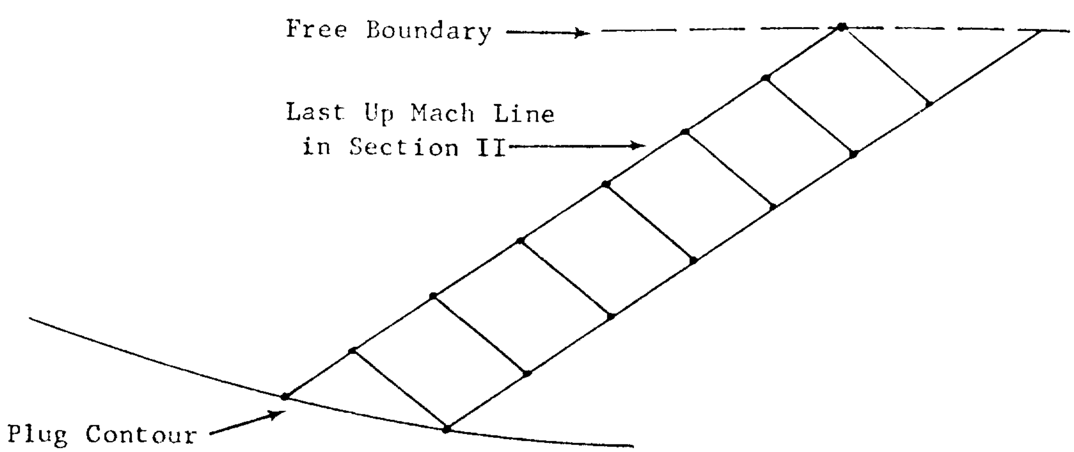
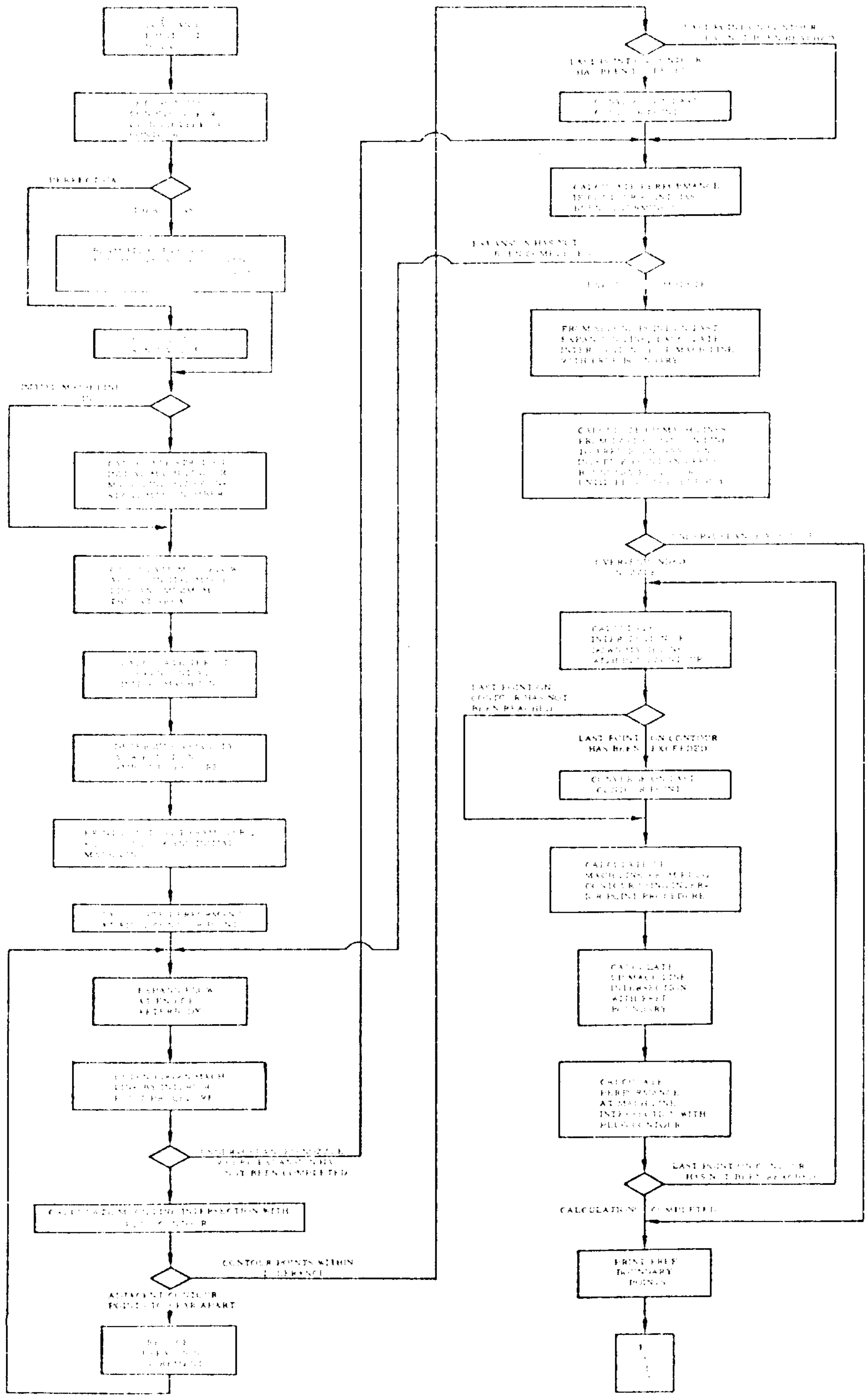


Figure 5

Single Expansion Plug Nozzle Performance



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C. SUBROUTINES

Since most of the subroutines are used many times in constructing a flow field or calculating performance, they are discussed individually in this section. The purpose of each subroutine, the equations used, and flow diagrams are given.

1. INTX Subroutine

Under certain conditions, the numerical solution of the characteristic system becomes difficult or impossible in determining the intersection of an up and a down Mach line. These conditions may occur if the slope of one or both of the Mach lines is extremely large or small. The problem can be eliminated by rotating the coordinate axis when solving the physical characteristics, and by modifying the axisymmetric term in the compatibility equations. The physical characteristic equations are invariant under this transformation. The INTX Subroutine performs the function of determining if rotation is needed, the form required for the axisymmetric term, and calls one of the following subroutines:

INT1 Subroutine - No rotation is used and the axisymmetric term of the compatibility equations for both up and down Mach lines use the differential dX.

INT2 Subroutine - The coordinate system is rotated and the axisymmetric term of the compatibility equations for both up and down Mach lines use the differential dX (for an up or down Mach line with very small slope).

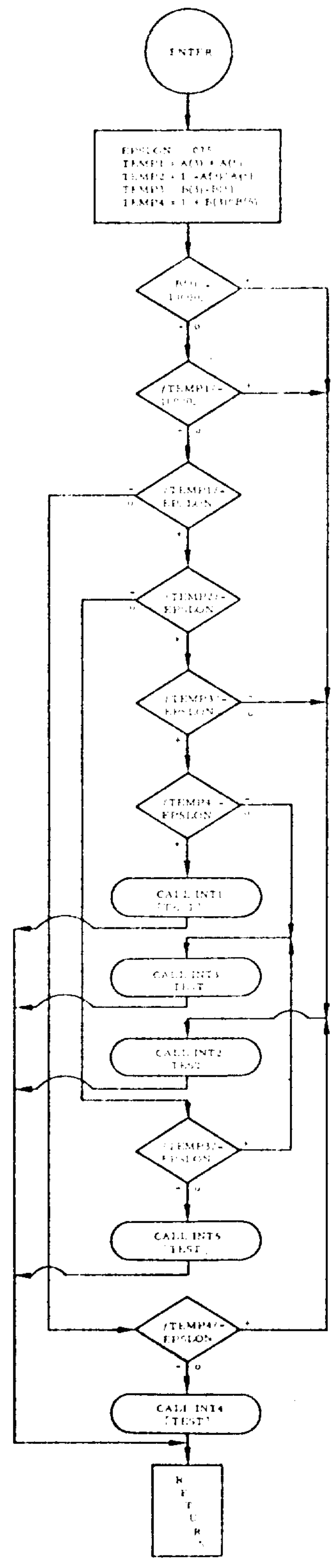
INT3 Subroutine - The coordinate system is rotated and the axi-symmetric term of the compatibility equations for both up and down Mach lines use the differential dY (for an up or down Mach line with very large slope).

INT4 Subroutine - The coordinate system is rotated and the axi-symmetric term of the up Mach line uses dX and the down Mach line dY (for an up Mach line with very small slope combined with a down Mach line with very large slope).

INT5 Subroutine - The coordinate system is rotated and the axi-symmetric term of the up Mach line uses dY and the down Mach line dX (for an up Mach line with very large slope combined with a down Mach line with very small slope).

The coordinates and flow conditions (W, tan θ, and tan α) at the points on an up and down Mach line must be stored into the variables A(I) and B(I), respectively.

Subroutine INTX



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2. INT1 Subroutine

Given the coordinates and flow conditions (W, tan θ, and tan α) at two points in the flow field not on the same Mach line, the INT1 Subroutine determines the coordinates and flow conditions at the intersection of an up Mach line from one point, and a down Mach line from the other. The known coordinates and flow conditions at points 1 and 2 must be stored into the variables A(I) and B(I), respectively. The corresponding properties at the interior point, 3, (figure 6) will be stored in the variable C(I).

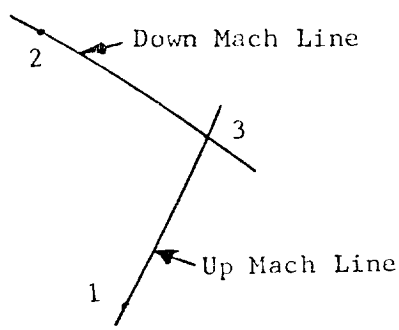


Figure 6

Written in finite difference form, the characteristic system is

$$(Y_3 - Y_1) = \left[\frac{\tan\theta + \tan\alpha}{1 - \tan\alpha\tan\theta}\right]_1 (X_3 - X_1)$$

and (2.1)

$$(Y_3 - Y_2) = \left[\frac{\tan\theta - \tan\alpha}{1 + \tan\alpha\tan\theta}\right]_2 (X_3 - X_2),$$

$$(W_3-W_1)\left[\frac{1}{W\tan\alpha}\right]_1 - (\tan\theta_3 - \tan\theta_1)\left[\frac{1}{1+\tan^2\theta}\right]_1 = \epsilon\left[\frac{\tan\alpha\tan\theta}{(1-\tan\alpha\tan\theta)Y}\right]_1 (X_3-X_1)$$

and (2.2)

$$(W_3-W_2)\left[\frac{1}{W\tan\alpha}\right]_2 + (\tan\theta_3 - \tan\theta_2)\left[\frac{1}{1+\tan^2\theta}\right]_2 = \epsilon\left[\frac{\tan\alpha\tan\theta}{(1+\tan\alpha\tan\theta)Y}\right]_2 (X_3-X_2).$$

The subscripts 1 and 2 indicate that the quantities in brackets are evaluated at these points.

Solving equations (2.1) simultaneously, the coordinates at point 3 are:

$$X_3 = \frac{Y_1 - \left[\frac{\tan\theta + \tan\alpha}{1 - \tan\alpha\tan\theta}\right]_1 X_1 - Y_2 + \left[\frac{\tan\theta - \tan\alpha}{1 + \tan\alpha\tan\theta}\right]_2 X_2}{\left[\frac{\tan\theta - \tan\alpha}{1 + \tan\alpha\tan\theta}\right]_2 - \left[\frac{\tan\theta + \tan\alpha}{1 - \tan\alpha\tan\theta}\right]_1}$$

and (2.3)

$$Y_3 = Y_2 + (X_3 - X_2)\left[\frac{\tan\theta - \tan\alpha}{1 + \tan\alpha\tan\theta}\right]_2 .$$

Flow conditions W_3 and $\tan\theta_3$ are then obtained from the simultaneous solution of equations (2.2).

$$W_3 = \frac{K_1 + K_2}{\left[\frac{1}{W\tan\alpha}\right]_2 \div \left[\frac{1}{1+\tan^2\theta}\right]_2 + \left[\frac{1}{W\tan\alpha}\right]_1 \div \left[\frac{1}{1+\tan^2\theta}\right]_1}$$

and (2.4)

$$\tan\theta_3 = K_2 + W_3\left[\frac{1}{W\tan\alpha}\right]_1 \div \left[\frac{1}{1+\tan^2\theta}\right]_1 ;$$

where:

$$K_1 = \tan\theta_2 + W_2\frac{\left[\frac{1}{W\tan\alpha}\right]_2}{\left[\frac{1}{1+\tan^2\theta}\right]_2} + \sigma\,(X_3 - X_2)\frac{\left[\frac{\tan\alpha\tan\theta}{(1+\tan\alpha\tan\theta)Y}\right]_2}{\left[\frac{1}{1+\tan^2\theta}\right]_2}$$

and

$$K_2 = \tan\theta_1 - W_1\frac{\left[\frac{1}{W\tan\alpha}\right]_1}{\left[\frac{1}{1+\tan^2\theta}\right]_1} + \sigma\,(X_1 - X_3)\frac{\left[\frac{\tan\alpha\tan\theta}{(1-\tan\alpha\tan\theta)Y}\right]_1}{\left[\frac{1}{1+\tan^2\theta}\right]_1} .$$

The tangent of the Mach angle ($\tan\alpha_3$), which is a function of W_3, is determined by the procedure described in TAGAL Subroutine for an ideal gas or by the procedure in the PRFCT Subroutine for a perfect gas.

Since the evaluation of equations (2.3) and (2.4) gives first approximations to X_3, Y_3, $\tan\theta_3$, W_3, and $\tan\alpha_3$, improved solutions are obtained by replacing the quantities in brackets with average values; that is,

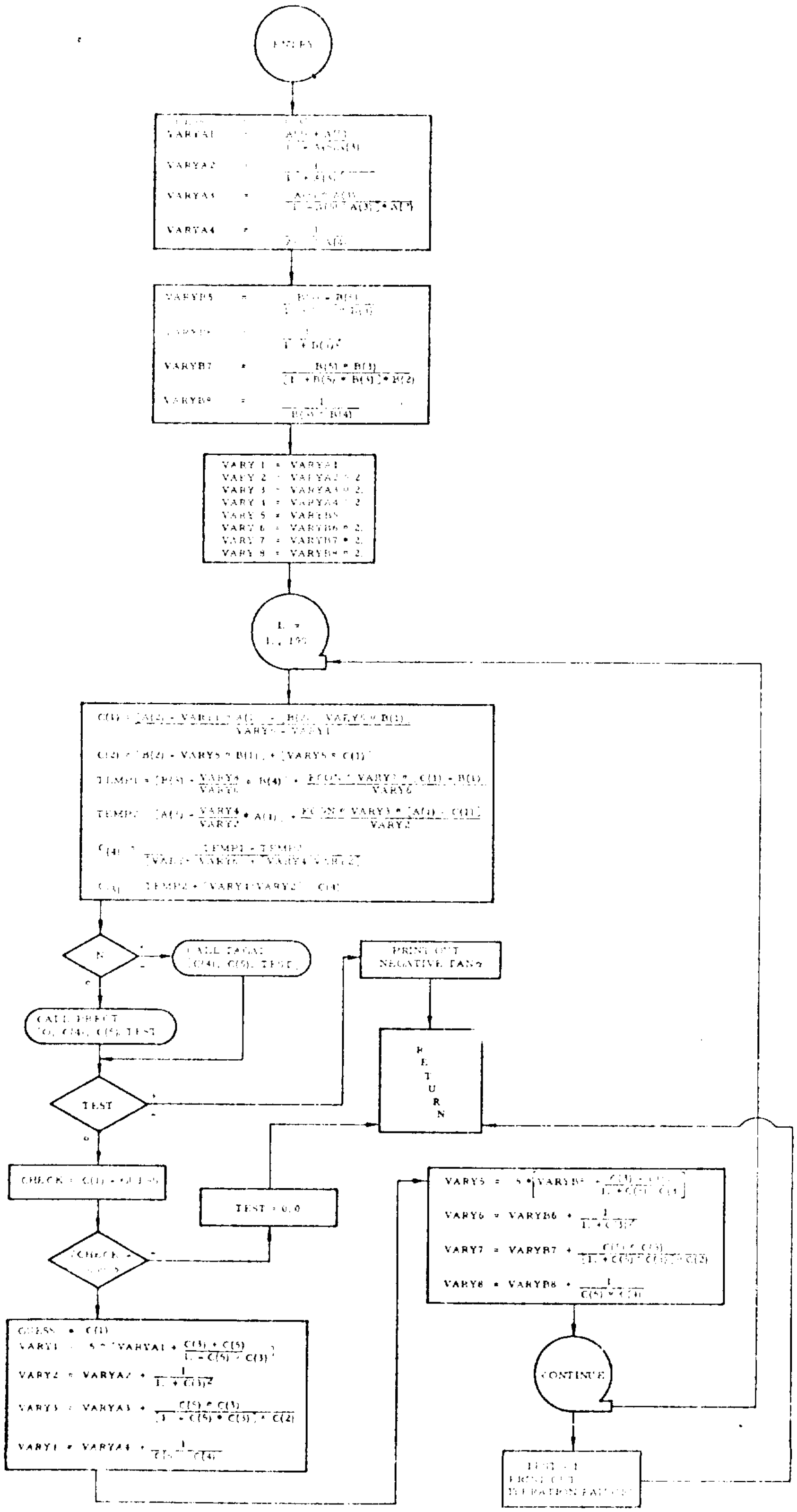
replace $\left[\dfrac{\tan\theta + \tan\alpha}{1-\tan\alpha\ \tan\theta}\right]_1$ by

$$\left[\frac{\tan\theta + \tan\alpha}{1-\tan\alpha\ \tan\theta}\right]_{1,3} = 1/2\left\{\left[\frac{\tan\theta + \tan\alpha}{1-\tan\alpha\ \tan\theta}\right]_1 + \left[\frac{\tan\theta + \tan\alpha}{1-\tan\alpha\ \tan\theta}\right]_3\right\}$$

This procedure for obtaining the improved solutions is repeated until successive values of X_3 are within a specified tolerance.

$$\left| X_3^i - X_3^{i-1} \right| \leq 0.000005$$

Subroutine INT 1



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3. INT2 Subroutine

Given the coordinates and flow conditions (W, tan θ, and tan α) at two points in the flow field not on the same Mach line, the INT2 Subroutine determines the coordinates and flow conditions at the intersection of an up Mach line from one point and a down Mach line from the other point, when the slope of either Mach line is very small. The known coordinates and flow conditions at points 1 and 2 must be stored into the variables A(I) and B(I), respectively. The corresponding properties at the intersection point, 3, (figure 7) will be stored in the variable C(I).

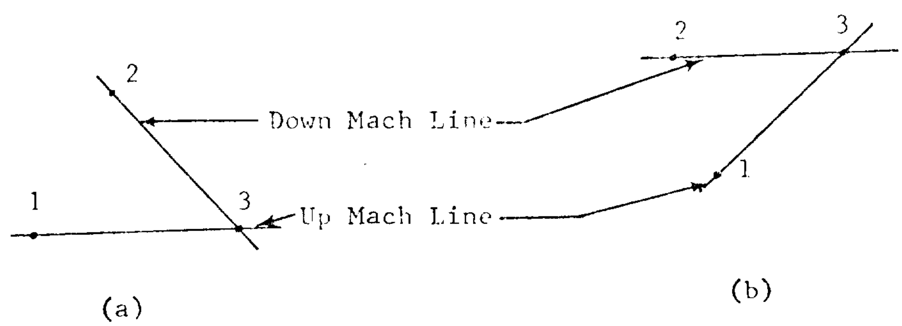


Figure 7

The characteristic system is the same as for the INT1 Subroutine (equations 2.1 and 2.2), except that the coordinate axes are rotated. The coordinate transformation is given by the following:

$$X' = X \cos \varphi + Y \sin \varphi$$

$$Y' = Y \cos \varphi - X \sin \varphi,$$

and (3.1)

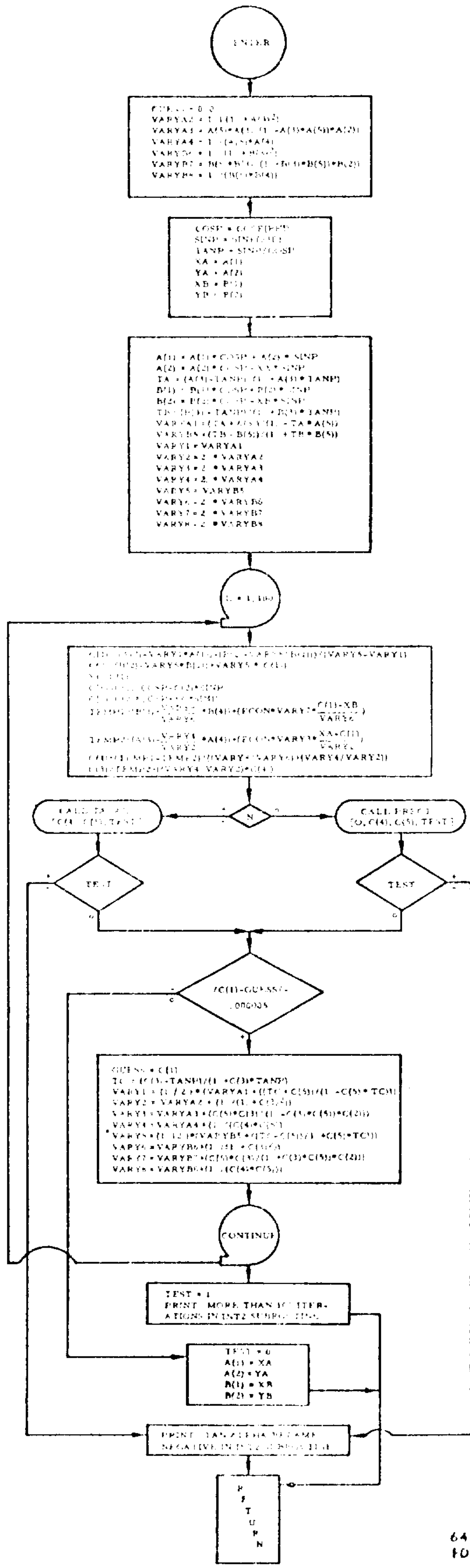
$$\tan \theta' = (\tan \theta - \tan \varphi) \div (1 + \tan \theta \tan \varphi)$$

where:

the prime indicates the rotated value and φ the angle of rotation.

Solving the physical characteristics (equation 2.1) simultaneously using the rotated values, the coordinates at point 3' are determined. The coordinate axes are then rotated back to their original position, and the flow conditions W_3 and tan θ_3 are obtained as in the INT1 Subroutine.

Subroutine INT2



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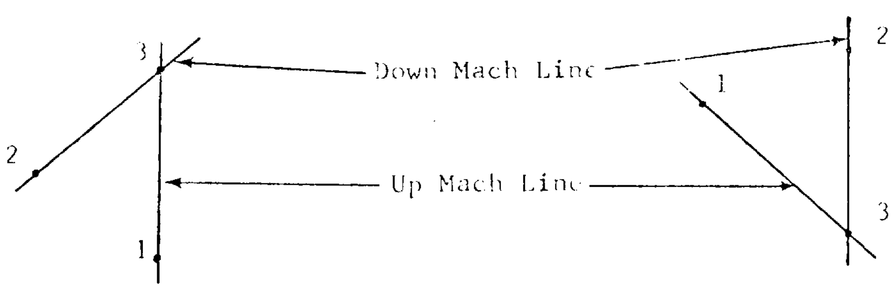
4. INT3 Subroutine

Given the coordinates and flow conditions (W, tan θ, and tan σ) at two points in the flow field not on the same Mach line, the INT3 Subroutine determines the coordinates and flow conditions at the intersection of an up Mach line from one point and a down Mach line from the other point, when the slope of either Mach line is very large. The known coordinates and flow conditions at points 1 and 2 must be stored into the variables A(I) and B(I), respectively. The corresponding properties at the intersection point, 3, (figure 8) will be stored in the variable C(I).



(a) (b)

Figure 8

The characteristic system is the same as for the INT1 Subroutine (equations 2.1 and 2.2), except that the axisymmetric term of the compatibility equations have the form

$$\sigma\left[\frac{\tan\sigma\ \tan\theta}{(\tan\theta + \tan\sigma)\ Y}\right]_1 (Y_3 - Y_1) \qquad (4.1)$$

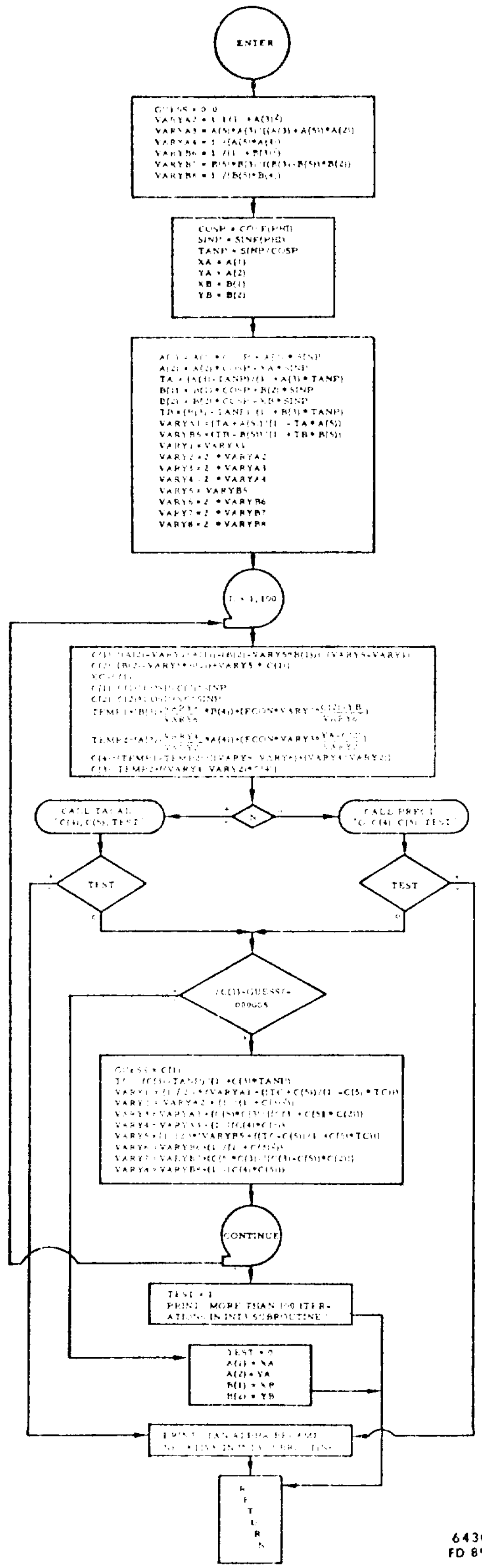
for an up Mach line, and

$$\sigma\left[\frac{\tan\sigma\ \tan\theta}{(\tan\theta - \tan\sigma)\ Y}\right]_2 (Y_3 - Y_2) \qquad (4.2)$$

for a down Mach line.

Because numerical solution of the physical characteristics is extremely difficult when the slope of a Mach line is very large, the coordinate axes must be rotated. The coordinate transformation and the calculation of flow conditions are the same as presented in the INT2 Subroutine.

Subroutine INT3



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5. INT4 Subroutine

Given the coordinates and flow conditions (W, $\tan\theta$, and $\tan\alpha$) at two points in the flow field not on the same Mach line, the INT4 Subroutine determines the coordinates and flow conditions at the intersection of an up Mach line from one point and a down Mach line from the other point, when the slope of the up Mach line is very small and the slope of the down Mach line very large. The known coordinates and flow conditions at points 1 and 2 must be stored into the variables A(I) and B(I), respectively. The corresponding properties at the intersection point, 3, (figure 9) will be stored in the variable C(I).

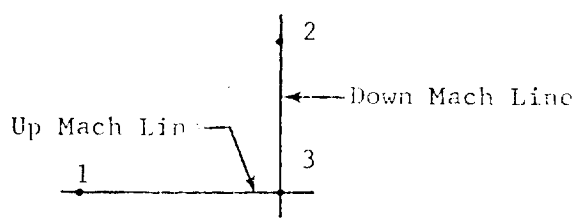


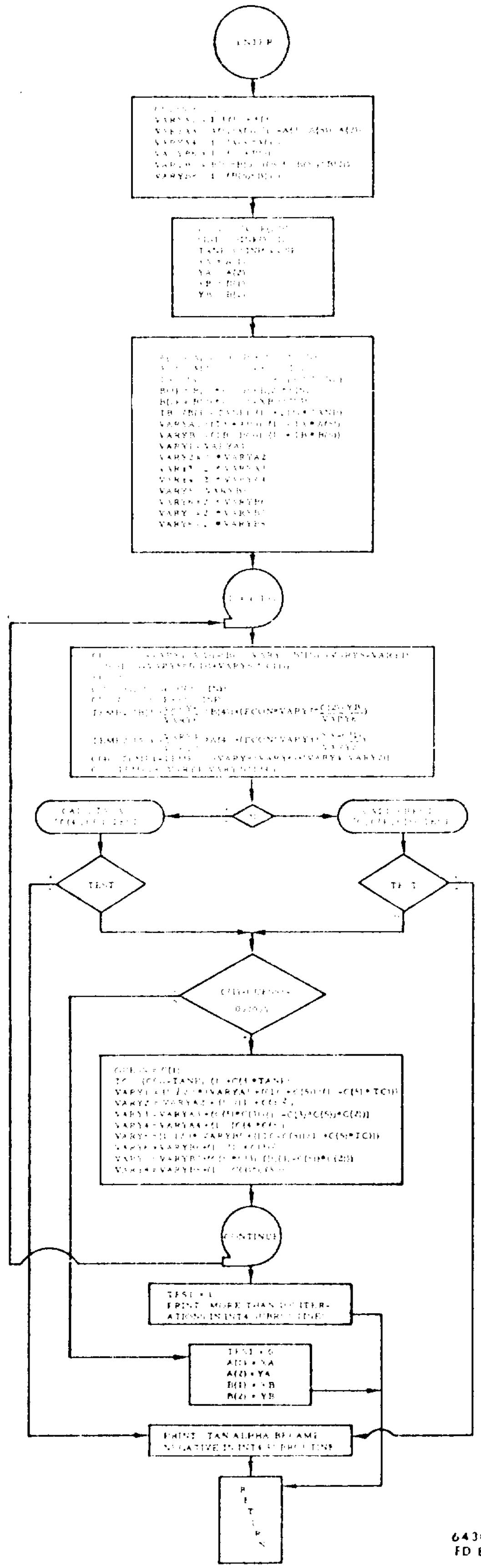
Figure 9

The characteristic system is the same as in INT1 Subroutine (equations 2.1 and 2.2), except that the axisymmetric term of the compatibility equation for the down Mach line is

$$\sigma \left[\frac{\tan\alpha \ \tan\theta}{(\tan\theta - \tan\alpha)\ Y} \right]_2 (Y_3 - Y_2). \qquad (5.1)$$

Because numerical solution of the physical characteristics is extremely difficult when the slope of a Mach line is large, the coordinate axes must be rotated. The coordinate transformation and the calculation of flow conditions are the same as presented in the INT2 Subroutine.

Subroutine INT4



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6. INT5 Subroutine

Given the coordinates and flow conditions (W, $\tan\theta$, and $\tan\alpha$) at two points in the flow field not on the same Mach line, the INT5 Subroutine determines the coordinates and flow conditions at the intersection of an up Mach line from one point and a down Mach line from the other point, when the slope of the up Mach line is very large and the slope of the down Mach line very small. The known coordinates and flow conditions at points 1 and 2 must be stored into the variables A(I) and B(I), respectively. The corresponding properties at the intersection point, 3, (figure 10)will be stored in the variable C(I).

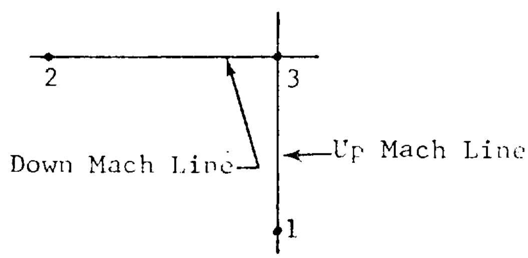


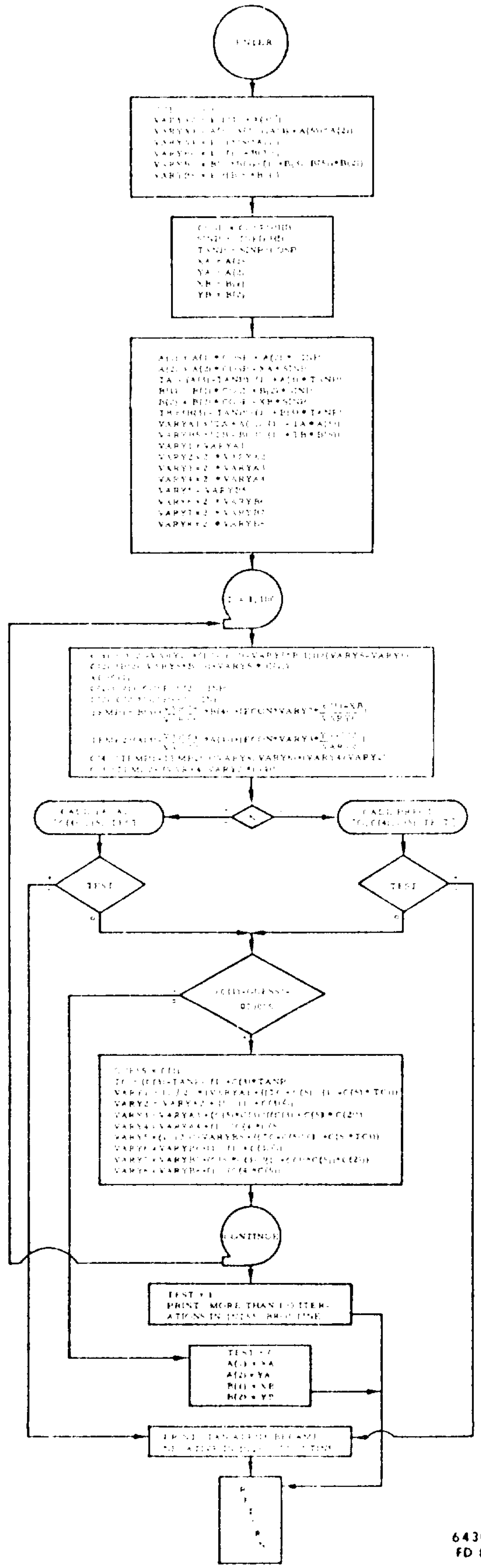
Figure 10

The characteristic system is the same for the INT1 Subroutine (equations 2.1 and 2.2), except that the axisymmetric term of the compatibility equation for the up Mach line is

$$\varepsilon \left[\frac{\tan\alpha \tan\theta}{(\tan\theta + \tan\alpha)Y}\right]_1 (Y_3 - Y_1). \qquad (6.1)$$

Because numerical solution of the physical characteristics is extremely difficult when the slope of a Mach line is large, the coordinate axes must be rotated. The coordinate transformation and the calculation of flow conditions are the same as presented in the INT2 Subroutine.

Subroutine INT5



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7. EXPAND Subroutine

The EXPAND Subroutine calculates the flow properties at a point after the flow has been expanded by an increment in the velocity ratio (figure 11). The slope, tan θ_C, of the expanded velocity ratio, W_C, is found

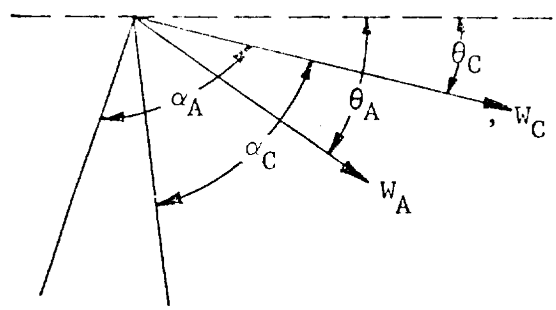


Figure 11

by integrating

$$\frac{d\ (\tan\ \theta)}{dW} = \frac{1 + \tan^2\ \theta}{W\ \tan\ \alpha} = f(W,\ \tan\ \theta). \tag{7.1}$$

Using the method of Runge-Kutta over ten intervals,

let

$$h = \frac{W_C - W_A}{10}$$

$$W_1 = W_A$$

and

$$\tan\ \theta_1 = \tan\ \theta_A .$$

Solve equations 7.2 to 7.8 as j = 1, 10.

$$K_1 = f(W_j,\ \tan\ \theta_j)h \tag{7.2}$$

$$K_2 = f(W_j + h/2,\ \tan\ \theta_j + K_1/2)h \tag{7.3}$$

$$K_3 = f(W_j + h/2,\ \tan\ \theta_j + K_2/2)h \tag{7.4}$$

$$K_4 = f(W_j + h,\ \tan\theta_j + K_3)h \tag{7.5}$$

$$\Delta(\tan\theta)' = 1/6\ (K_1 + 2\ K_2 + 2\ K_3 + K_4) \tag{7.6}$$

$$\tan\theta_{j+1} = \tan\theta_j + \Delta(\tan\theta) \tag{7.7}$$

$$W_{j+1} = W_j + h \tag{7.8}$$

The properties corresponding to W_C are

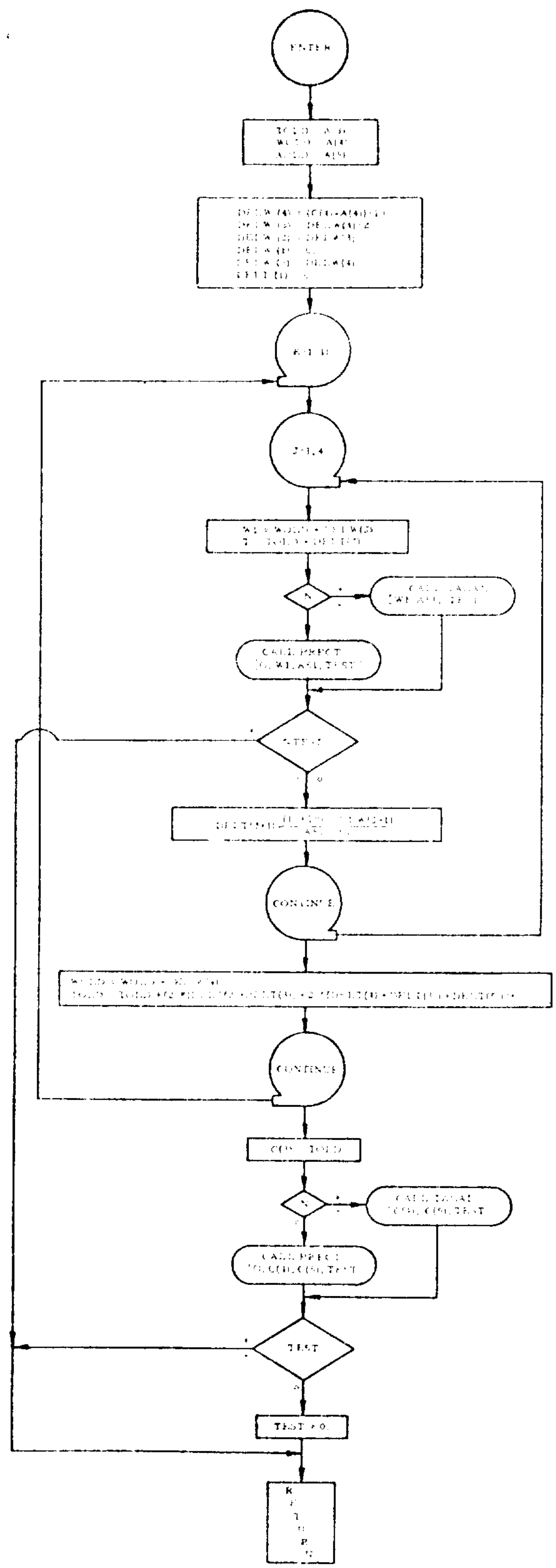
$$W_C = W_{11}$$

$$\tan\alpha_C = f(W_C)$$

$$\tan\theta_C = \tan\theta_{11},$$

and are stored in the variable C(I). Since the expansion occurs about a sharp corner, the X and Y coordinates remain unchanged.

Subroutine EXPAND



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8. BOUNDD Subroutine

Given the coordinates and flow conditions (W, tan θ, and tan α) at point 1 (figure 12) near a physical boundary, the BOUNDD Subroutine determines the corresponding values at a boundary point 3, which is the intersection of the down Mach line from point 1 with the contour of the nozzle.

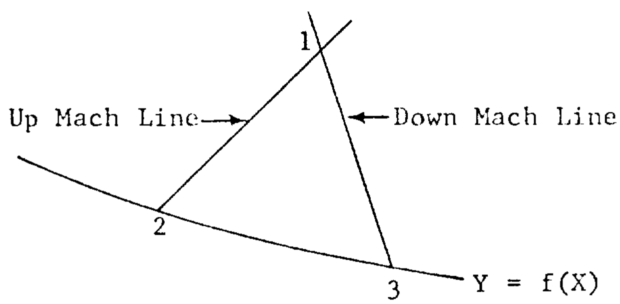


Figure 12

The coordinates and flow conditions at the interior point 1 and the previous boundary point 2 are stored in variables B(I) and A(I), respectively. The calculated values at point 3 are stored into variable C(I).

The boundary is represented by the equation Y = f(X), and is evaluated in the EVAL Subroutine. The necessary first guess on Y_3 is the Y-coordinate of the last calculated point on the contour.

An iteration on equation (8.1) is required to determine the coordinates X_3 and Y_3.

$$X_3 = X_1 + \frac{Y_3 - Y_1}{\left[\frac{\tan\theta - \tan\alpha}{1 + \tan\alpha \tan\theta}\right]_1} \qquad (8.1)$$

$$Y_3 = f(X_3)$$

After two successive values of Y_3 are within a tolerance of 0.000005, the flow conditions are calculated by

$$\tan\theta_3 = \frac{d}{dX} f(X_3)$$

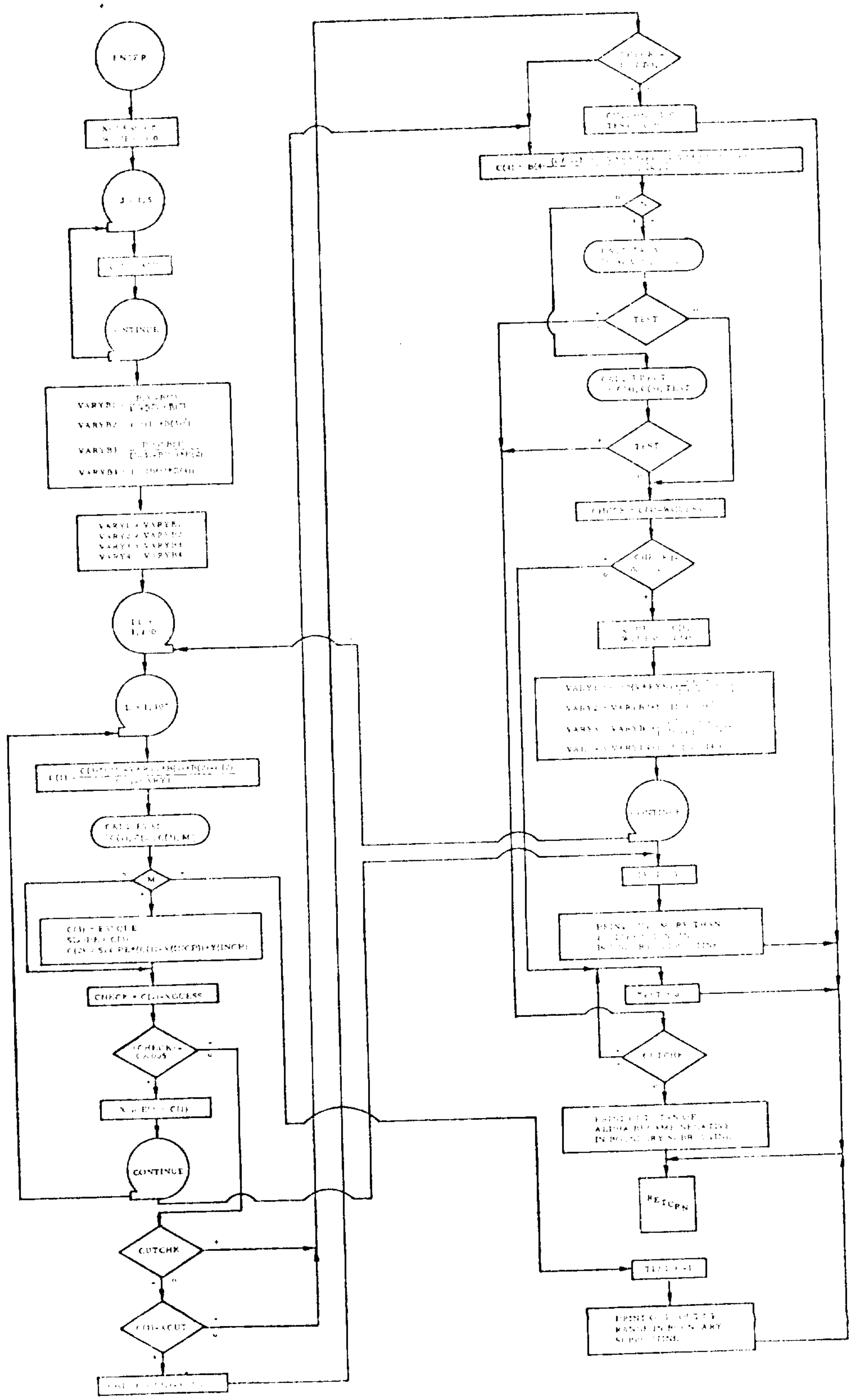
$$W_3 = W_1 - \frac{\left[\frac{1}{1+\tan^2\theta}\right]_1 (\tan\theta_3 - \tan\theta_1) - \sigma\left[\frac{\tan\alpha\tan\theta}{(\tan\theta - \tan\alpha)Y}\right]_1 (Y_3 - Y_1)}{\left[\frac{1}{W\tan\alpha}\right]_1} .$$

Tan α_3 is a function of W_3, and is calculated in TAGAL Subroutine for an ideal gas or in PRFCT Subroutine for a perfect gas.

Improved solutions are obtained by replacing the quantities in brackets with average values as described in the INT1 Subroutine. The procedure is repeated until

$$\left| W_3^i - W_3^{i-1} \right| \leq 0.000005.$$

Subroutine BOUNDD



9. FREE Subroutine

Given the coordinates and flow conditions (W, tan θ, and tan α) at point 1 (figure 13) near a free boundary, the FREE Subroutine determines the corresponding values at a boundary point 3, which is the intersection of the up Mach line from point 1, and a boundary of constant pressure (or velocity).

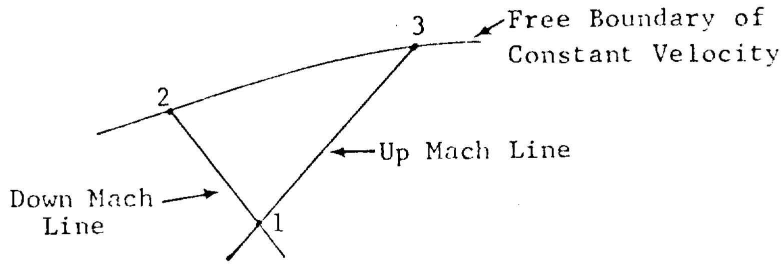


Figure 13

The coordinates and flow conditions at the interior point 1 and the previous boundary point 2 are stored into variables A(I) and B(I), respectively. The calculated values at point 3 are stored into the variable C(I).

Since W and tan α are constant along the free boundary, these parameters are set equal to the values at point 2. The coordinates at point 3 are calculated by

$$X_3 = \frac{Y_2 - Y_1 + X_1 \left[\frac{\tan\theta + \tan\alpha}{1 - \tan\alpha \tan\theta}\right]_1 - X_2 \left[\tan\theta\right]_2}{\left[\frac{\tan\theta + \tan\alpha}{1 - \tan\alpha \tan\theta}\right]_1 - \left[\tan\theta\right]_2} \tag{9.1}$$

and

$$Y_3 = Y_2 + (X_3 - X_2) \left[\tan\theta\right]_1 .$$

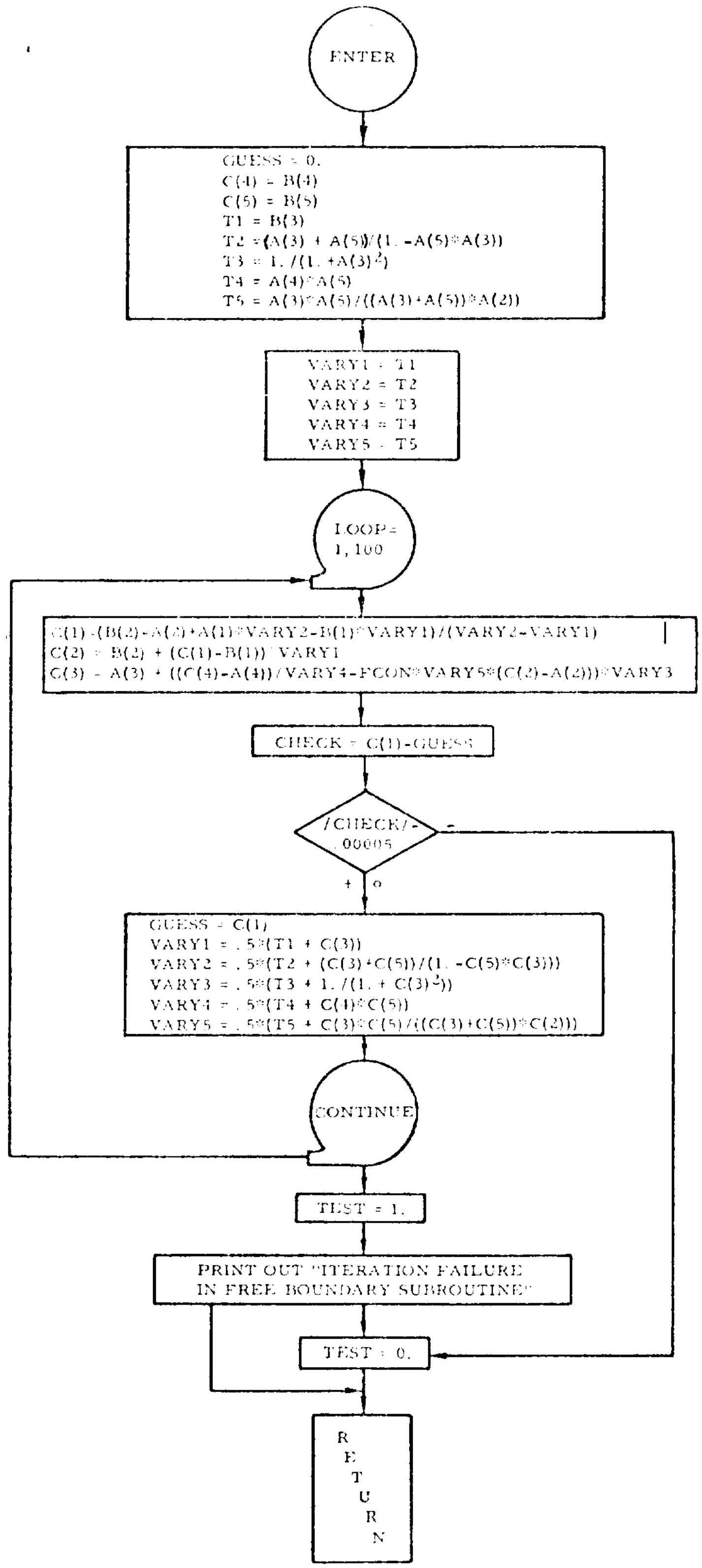
The slope of the free boundary is calculated by

$$\tan \theta_3 = \tan \theta_1 + \left[\frac{1}{1+\tan^2\theta}\right]_1 \left\{\frac{W_3 - W_1}{\left[W \tan \alpha\right]_1} - \left[\frac{\tan \alpha \tan \theta}{(\tan \theta - \tan \alpha)Y}\right]_1 (Y_3 - Y_1)\right\}.$$

Improved solutions are obtained by replacing the quantities in brackets with average values as described in the INT1 Subroutine. The procedure is repeated until

$$\left| X_3^i - X_3^{i-1} \right| \leq 0.00005.$$

Subroutine FREE



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10. TMLINE Subroutine

Depending on the value of TF, the TMLINE Subroutine calculates a starting down Mach line that has either a constant Mach number or a constant slope (a straight line). The straight Mach line is limited to a perfect gas model. For both options the initial coordinates along the contour must be stored in X(1) and Y(1); the calculated Mach line with NUM points is stored in BL(I,J), beginning at the expansion point at the end of the vehicle afterbody.

The following procedure is used to develop a down Mach line of constant Mach number from the vehicle afterbody to the plug contour. As a first approximation, the initial coordinates of the contour are used as the intersection of the starting line with the contour. Since the Mach number is constant along the starting line, the values of $\tan \alpha$ and W also remain constant.

Using the method of Runge-Kutta for two first-order differential equations, the Y coordinate and $\tan \theta$ for additional points on the Mach line are calculated by integrating

$$\frac{dY}{dX} = \frac{\tan \theta - \tan \alpha}{1 + \tan \alpha \tan \theta},$$

and

$$\frac{d(\tan \theta)}{dX} = \frac{\tan \alpha \tan \theta \,(1 + \tan^2 \theta)}{Y\,(1 + \tan \alpha \tan \theta)}.$$

If, after calculating the intersection of the starting line with the Y-axis, this point is not within ± 0.0000001 of the value of 1.0, the initial point of the starting line must be shifted along the contour and a new Mach line constructed. This procedure is continued until the above tolerance is met.

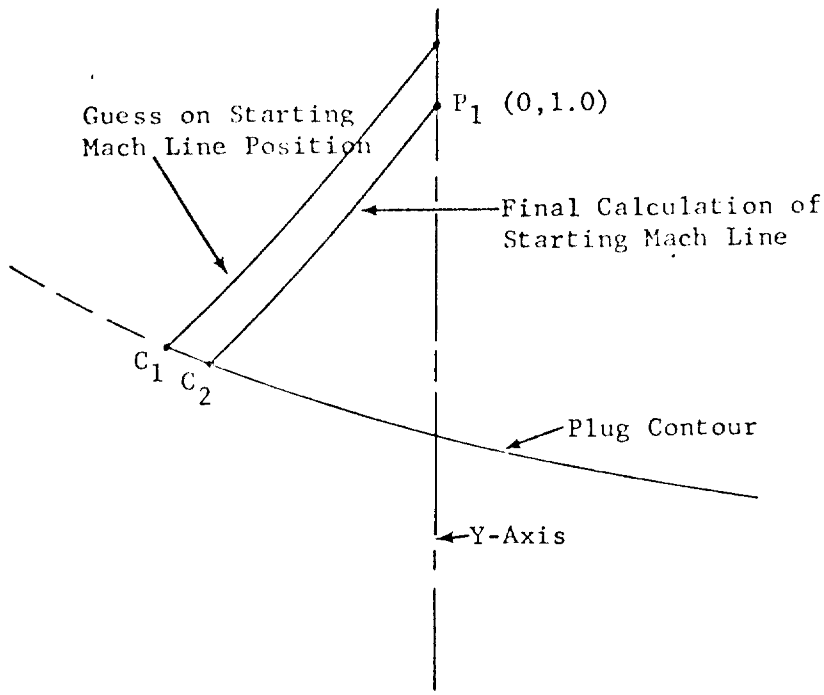


Figure 14

For a straight down Mach line from the expansion point at the end of the vehicle afterbody to the first input point on the plug contour, the slope of the Mach line is

$$\frac{dY}{dX} = \frac{1.0 - Y_1}{0.0 - X_1} \quad .$$

Since the slope, tan θ, at the first point on the contour is known, the Mach angle and velocity ratio, tan α and W, at that point can be calculated by

$$\tan \alpha = \frac{\tan \theta - dY/dX}{1 + (dY/dX) \tan \theta} \quad ,$$

and

$$W = f(\tan \alpha) \quad .$$

If tan α is negative, the first point on the contour is shifted to the right along the contour by DELTX until tan α is positive. In increments of ΔX, additional points along the straight Mach line are calculated by the following procedure.

$$X_{i+1} = X_i + \Delta X$$

Using the method of Runge-Kutta, tan α is calculated by integrating

$$\frac{d(\tan \alpha)}{dX} = \frac{\tan \alpha \left(\frac{dY}{dX} + \tan \alpha\right)}{\left(X \frac{dY}{dX} + 1\right)\left[1 + \frac{1 - (\gamma+1/\gamma-1)}{1 + (\gamma+1/\gamma-1) \tan^2 \alpha}\right]}$$

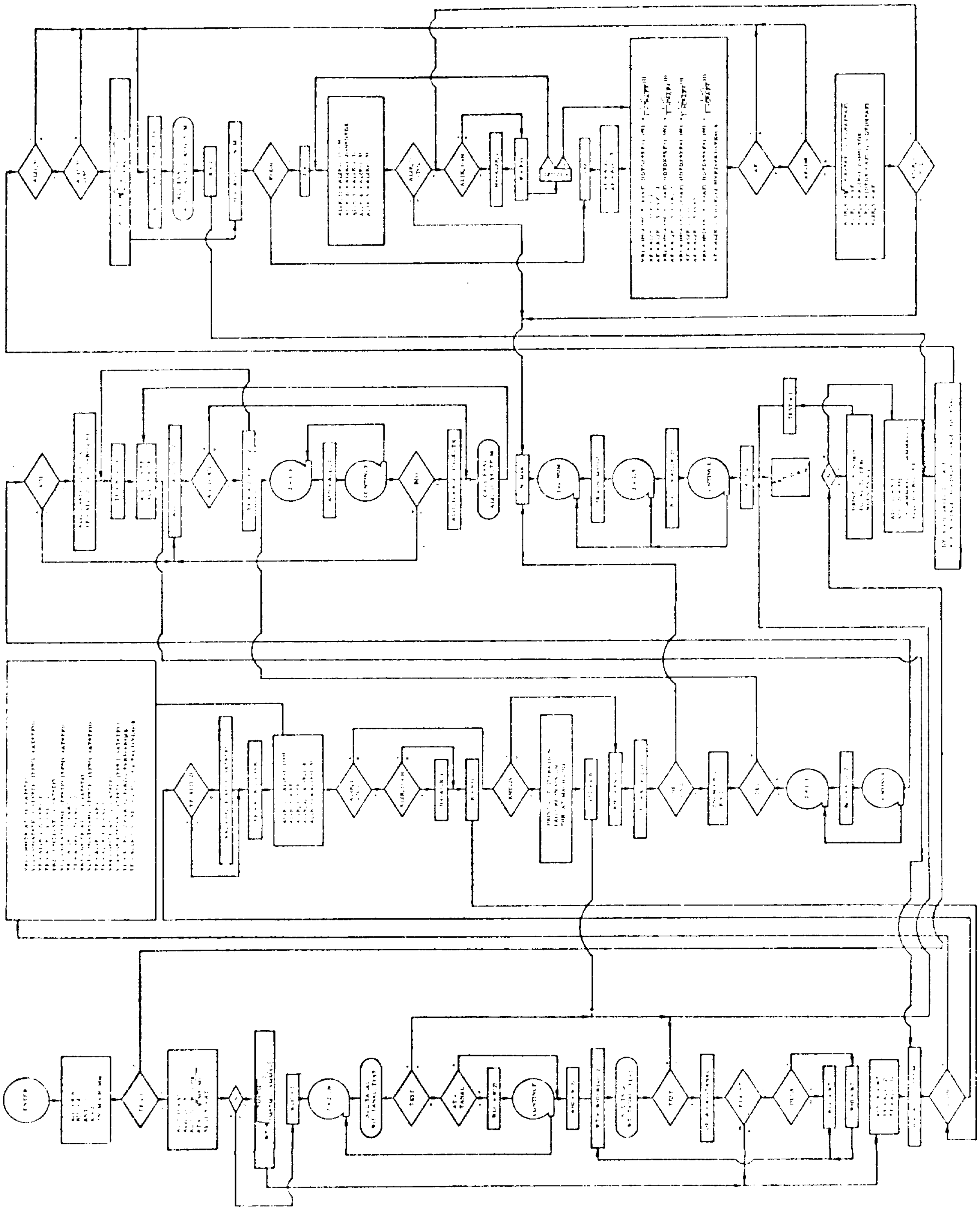
The Y coordinate, tan θ, and velocity ratio are calculated by the following equations:

$$Y_{i+1} = Y_i + \Delta X \frac{dY}{dX},$$

$$\tan \theta = \frac{\frac{dY}{dX} + \tan \alpha}{1 - \frac{dY}{dX} \tan \alpha}, \text{ and}$$

$$W = \sqrt{\frac{1 + \tan^2 \alpha}{\frac{\gamma+1}{\gamma-1} \tan^2 \alpha + 1}}$$

Subroutine TMLINE



11. PRFCT Subroutine

The PRFCT Subroutine is made up of four perfect gas relationships, which are a function of velocity ratio and a constant specific heat ratio. Depending on the value of the parameter (L), this subroutine calculates either tan α, ratio of static to total density, ratio of static to total pressure, or Mach number.

The following equations are evaluated by the PRFCT Subroutine.

$$\tan \alpha = \sqrt{\frac{1 - W^2}{W^2 \left(\frac{\gamma + 1}{\gamma - 1}\right) - 1}} \qquad (L = 0)$$

$$\rho/\rho_o = [1 - W^2]^{\frac{1}{\gamma - 1}} \qquad (L = 1)$$

$$P/P_o = [1 - W^2]^{\frac{\gamma}{\gamma - 1}} \qquad (L = 2)$$

$$M = \sqrt{\frac{W^2 \left(\frac{2}{\gamma - 1}\right)}{1 - W^2}} \qquad (L = 3)$$

The following is an explanation of the subroutine call list.

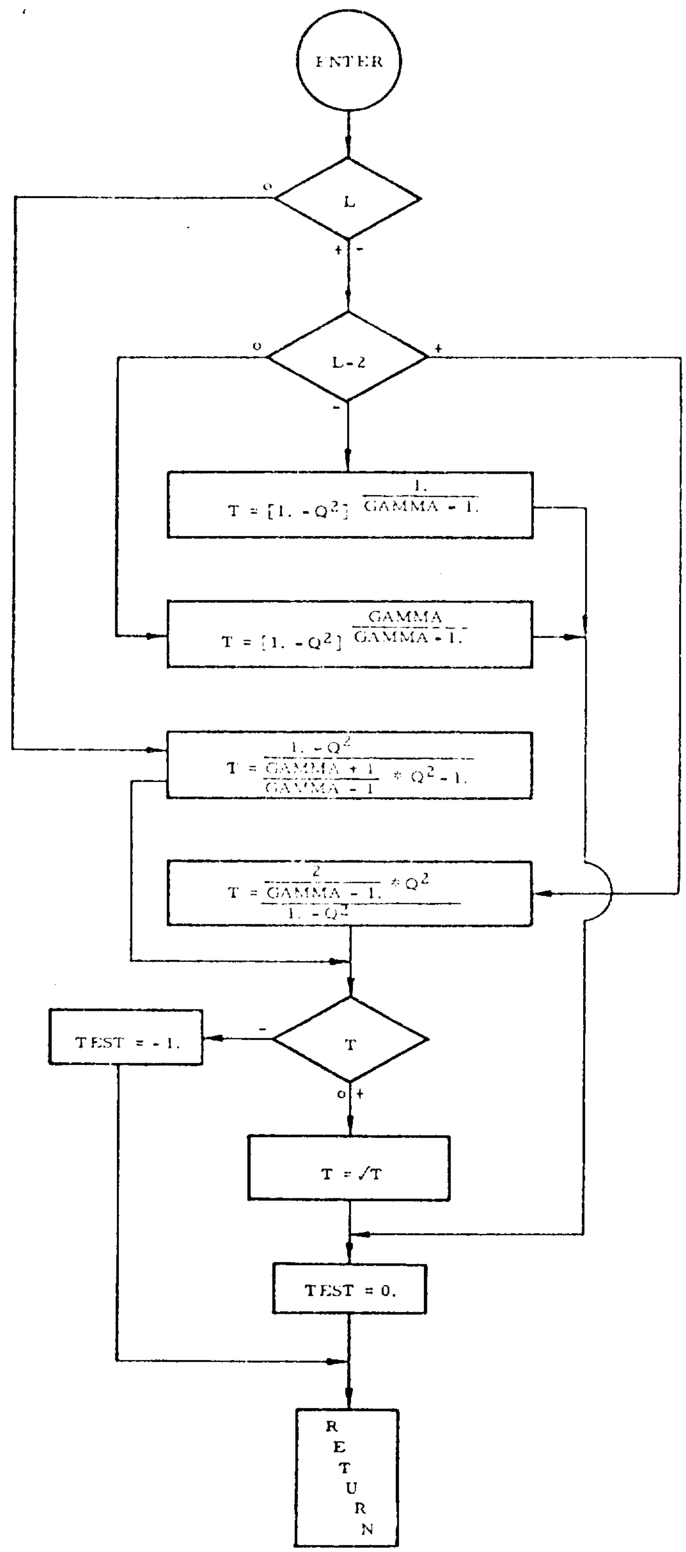
L - Indicates parameter to be calculated

Q - The known or input value of velocity ratio V/V_{max}

T - Variable that will contain the calculated value

TEST - An error signal in case of a subsonic velocity.

Subroutine PRFCT



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12. TAGAL Subroutine

For an ideal gas, the TAGAL Subroutine is used to calculate tan α as a function of a known velocity ratio ($W = V/V_{sonic}$). For the option where the local frozen sound speeds from the table of gas properties are not used,

$$\tan \alpha = \sqrt{\frac{1}{V_{sonic}^2 \quad W^2 \left(\frac{d\rho/dW}{dP/dW}\right) - 1}} \quad .$$

A beam fit evaluation of the gas properties is necessary to determine the values of $d\rho/dW$ and dP/dW.

If the local frozen sound speed option is used, then

$$\tan \alpha = \sqrt{\frac{1}{[V_{sonic} \quad W/c]^2 - 1}} \quad ,$$

where the local frozen sound speed (c) is also determined from a beam fit evaluation of the gas properties table.

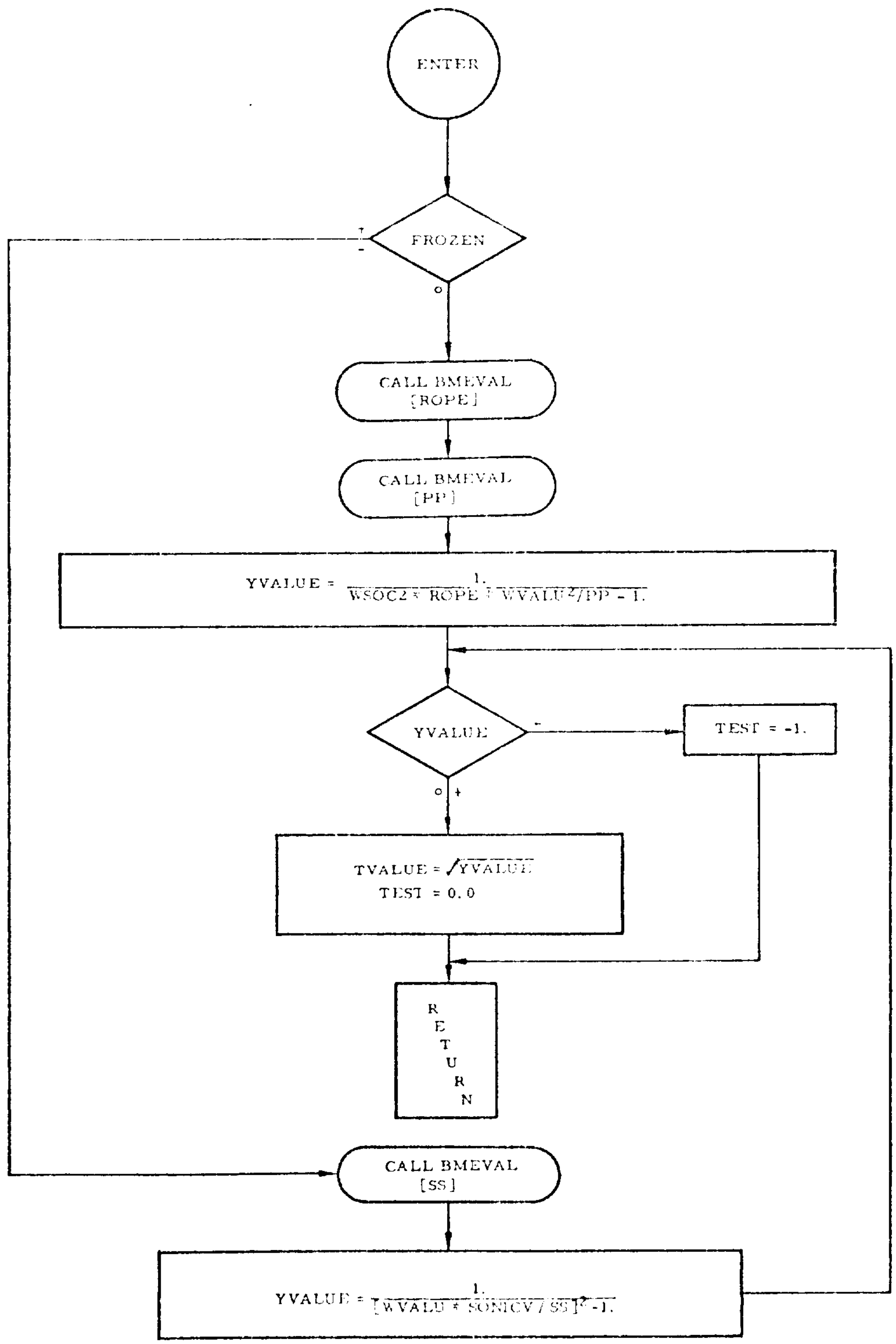
The following is an explanation of the subroutine call list.

WVALU - The known value of velocity ratio

TVALU - Value of tan α corresponding to WVALU

TEST - A signal that the input velocity ratio is subsonic. If TEST = -1, subsonic; TEST = 0, supersonic.

Subroutine TAGAL



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13. SONICP Subroutine

For an ideal gas, the SONICP Subroutine is called to adjust the units, determine the sonic values, and "beam fit" gas properties. Corresponding values of specific impulse, $\frac{\text{lbf sec}}{\text{lbm}}$; density, lbm/ft^3 ; pressure, (lbf/in^2); and local frozen sound speed, (ft/sec), must be stored into variables W(I), RO(I), P(I), and VS(I), respectively. The subroutine converts the units of pressure to lbf/ft^2 and density to $\frac{\text{lbf sec}^2}{\text{ft}^4}$.

If the program is to calculate local sound speeds, the pressure and density is beam fit as a function of specific impulse to calculate the sonic velocity at the throat. The sonic I_s is first bracketed by two values of specific impulse and a halving process is used until

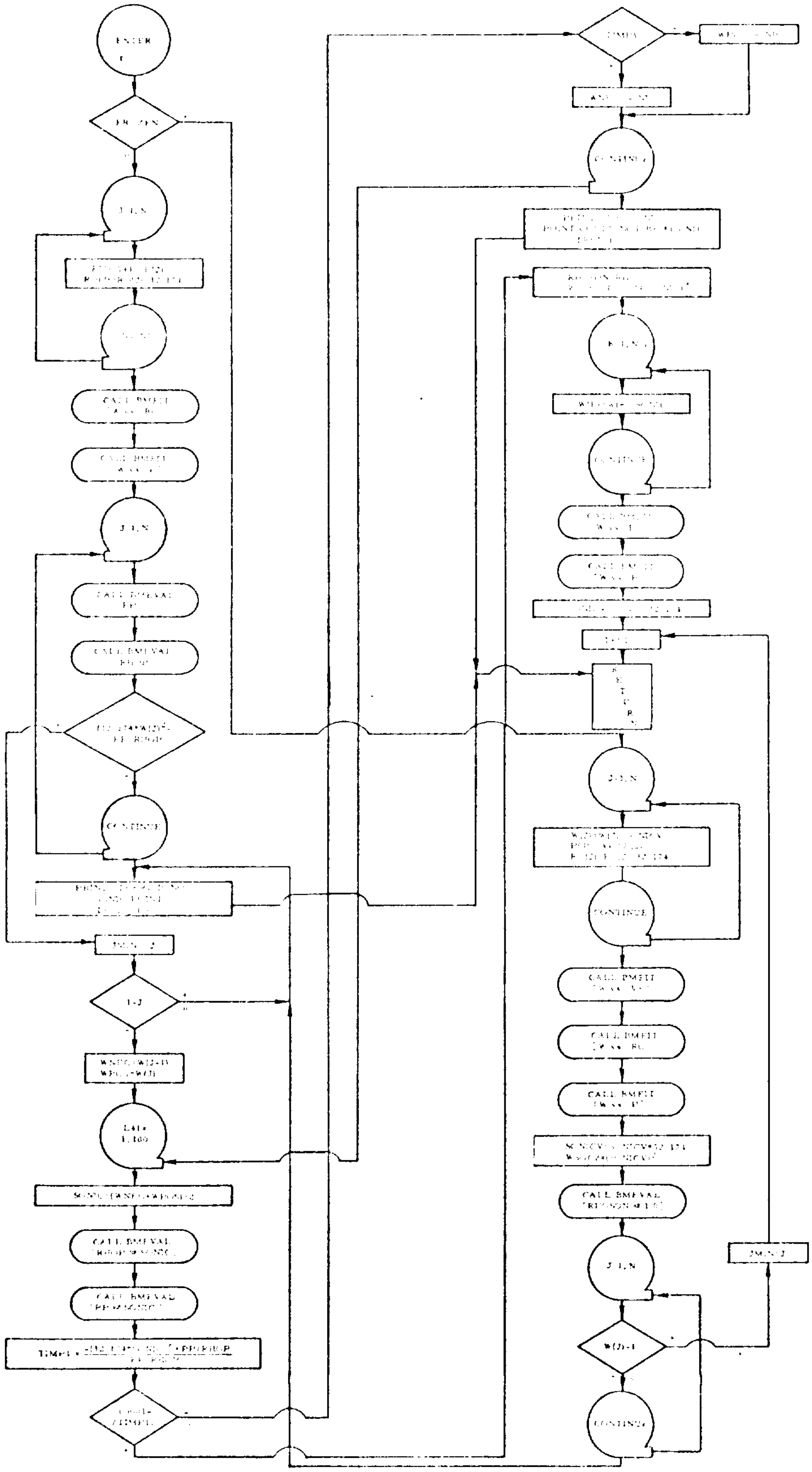
$$\frac{dP/dI_s}{d\rho/dI_s} = V^2_{sonic} \pm 0.00001,$$

where:

V_{sonic}/g_0 is the value of I_s at which dP/dI_s and $d\rho/dI_s$ are evaluated. The velocity and density at this point are stored into variables SONICV and RHOSON, respectively. All of the specific impulse values are converted to velocity ratios by dividing each one by the sonic I_s. The pressure and density is then beam fit again as a function of velocity ratio.

For the option where the local frozen sound speeds are used, the curve of c vs W is beam fit for the purpose of evaluating the local speed of sound throughout the flow field. Also, the iteration to determine the sonic velocity at the throat is eliminated because this value is given in the input.

Subroutine SONICP



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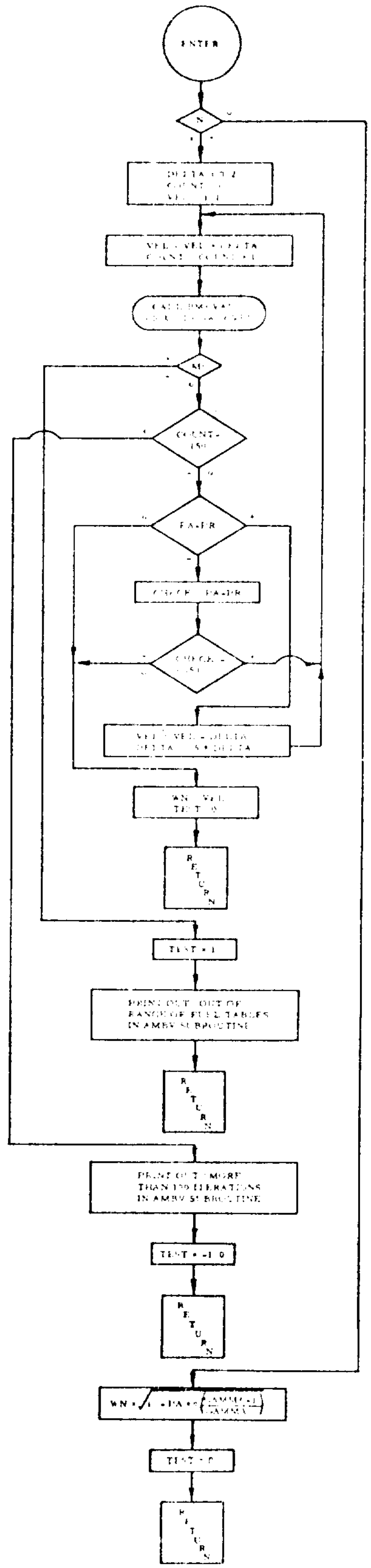
14. AMBV Subroutine

For an ideal gas, the AMBV Subroutine determines the velocity ratio corresponding to an input value of pressure (PA). From the beam fit of P vs W, PA is bracketed by evaluating the curve at increasing values of W. A halving process is used to determine the value of the velocity ratio corresponding to PA and stored in WN.

Subroutine AMBV



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15. TMFLOW Subroutine

The TMFLOW Subroutine calcuates the mass flow across the starting down Mach line to determine throat area (A*). The starting Mach line must be stored in the two-dimensioned variable BL(I,J), and the total mass flow through the nozzle is stored into XMFLOW.

To calculate the mass flow for an axisymmetric nozzle, the following equation is integrated along the starting down Mach line.

$$\dot{\omega} = \int \left(\rho \; W \tan \alpha \frac{\sqrt{1 + \tan^2 \theta}}{\tan \theta - \tan \alpha} \; 2\pi Y \right) dY \tag{15.1}$$

For two-dimensional flow, the $2\pi Y$ term is omitted.

The increment of mass flow between the first two points on the Mach line is calculated by trapezoidal integration. In the above equation, let Q represent the quantity in parenthesis; then the first mass flow increment is found by

$$\dot{\omega}_2 = \left(\frac{Q_1 + Q_2}{2} \right) (X_2 - X_1) \; .$$

The remaining increments of mass flow, as illustrated in figure 15, are determined by the parabolic integration of equation (15.1) for each point on the starting down Mach line.

$$\dot{\omega}_{i+1} = \dot{\omega}_i + \Delta X_{i+1} \left[\frac{1}{2}(Q_i + Q_{i+1}) - \frac{1}{6} \left(\frac{\Delta Y_{i+1}}{\Delta Y_i} \right) \left(\frac{\Delta Y_i \cdot \Delta Q_{i+1} - \Delta Y_{i+1} \cdot \Delta Q_i}{\Delta Y_{i+1} + \Delta Y_i} \right) \right] .$$

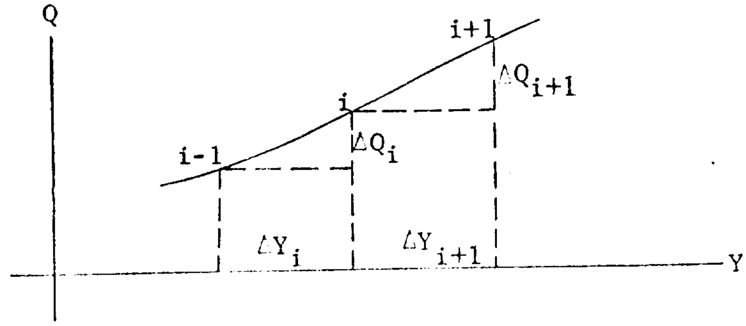
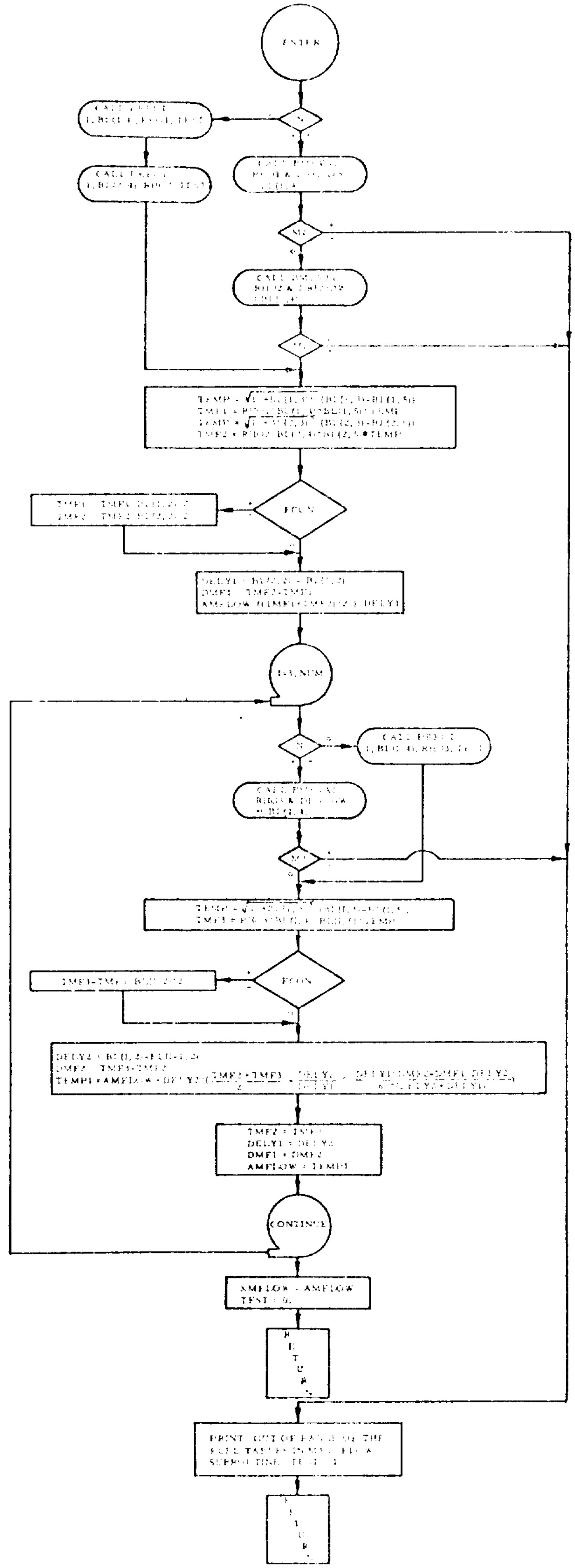


Figure 15

Subroutine TMFLOW



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16. CTGT Subroutine

The gross thrust coefficient at the first contour point is calculated by the CTGT Subroutine. In this subroutine, the starting down Mach line is expected to be stored in BL(I,J), and the calculated thrust coefficient will be stored in the variable CTG.

For a perfect gas and axisymmetric flow, the gross thrust coefficient is determined by integrating the following equation along the starting Mach line:

$$C_{TG_T} = \int \frac{1}{A^*} \left[\frac{\tan \alpha}{\tan \alpha - \tan \theta} \quad M^2 \; \gamma + 1 \right] \frac{P}{P_o} \quad 2\pi Y \; dY.$$

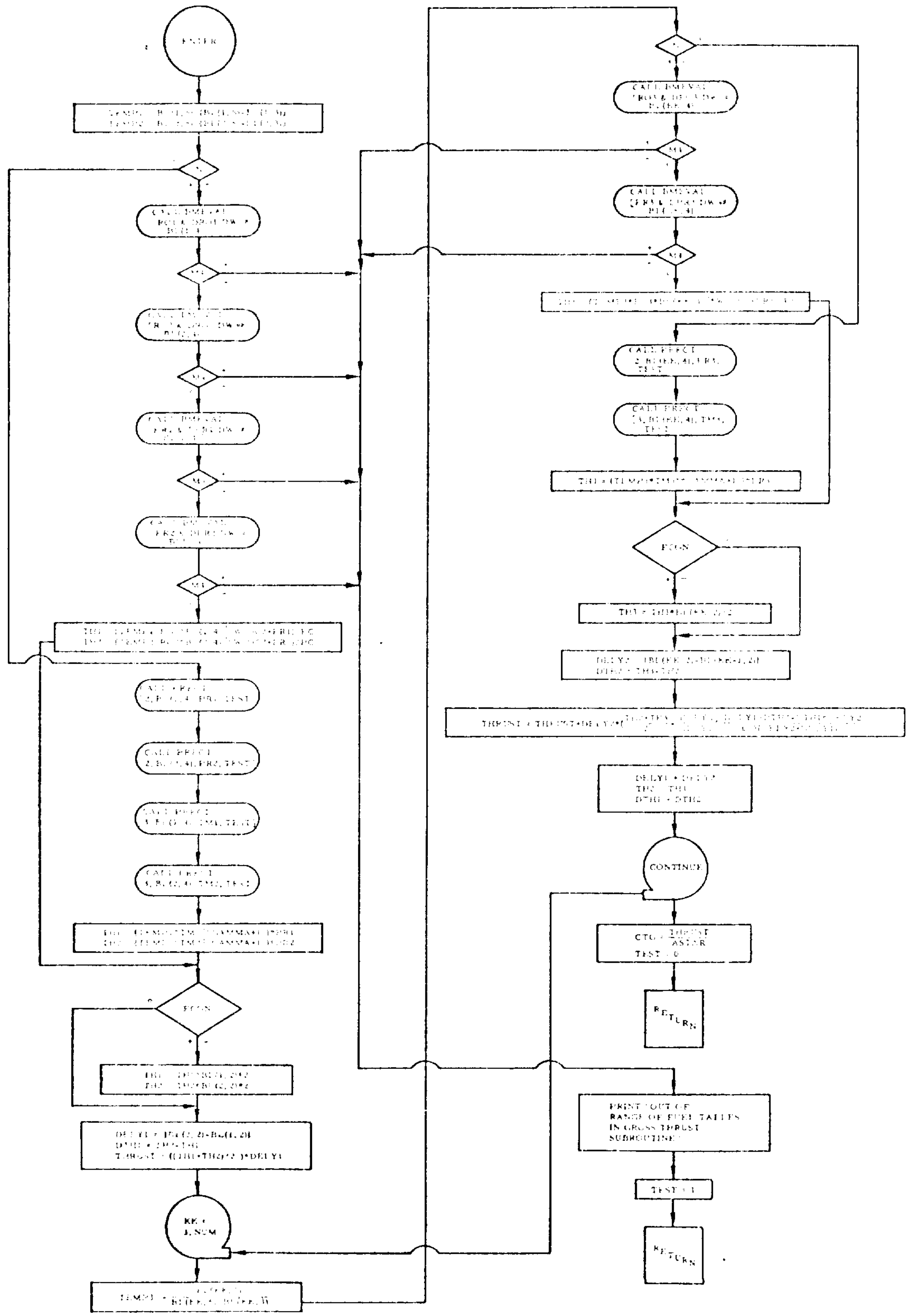
For an ideal gas and axisymmetric flow,

$$C_{TG_T} = \int \left[\frac{\dfrac{\tan \alpha}{\tan \alpha - \tan \theta} \; \rho \; W^2 \; V^2_{sonic} + P}{A^* \; P_C} \right] 2\pi Y \; dY.$$

For either gas model and two-dimensional flow, the $2\pi Y$ term in the above equations are omitted.

The same integration procedure as explained in the TMFLOW Subroutine is used for solving the proceeding equations; that is, use trapezoidal integration for the first increment on the Mach line and parabolic integration for the remaining increment.

Subroutine CTGT



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17. PERFOR Subroutine

After the flow conditions at each Mach line intersection with the contour have been determined and these values stored into variable C(I), the PERFOR Subroutine is called to calculate and print the performance at this point. For parameters that require integration, trapezoidal integration is used for the first intersection, and parabolic integration is used for the remaining points. The following are the nine parameters printed at each contour intersection.

PRF(1) The ratio of the X coordinate to the throat radius as stored in C(I)

PRF(2) The ratio of the Y coordinate to the throat radius as stored in C(2)

PRF(3) The slope of the contour or tan θ as stored in C(3)

PRF(4) The Mach number calculated by $M = \sqrt{1 + \frac{1}{\tan^2 \alpha}}$, where $\tan \alpha = C(5)$

PRF(5) The ratio of static pressure to chamber or total pressure, a function of W = C(4)

PRF(6) The ratio of specific heats that is input for a perfect gas, but for an ideal gas $\gamma = c^2\ \rho/P$

PRF(7) The ratio of accumulated surface area to the throat area (A*)

For two-dimensional flow, $A_s/A^* = 1/A^* \int ds$

For axisymmetric flow, $A_s/A^* = 2\pi/A^* \int Y ds$

where: $ds = \sqrt{(dX)^2 + (dY)^2}$

PRF(8) The gross thrust coefficient where the value at the first point is stored in CTG

For axisymmetric flow, $CTG = CTG + \int \frac{P}{P_o \cdot A^*} 2\pi Y \, dY$

For two-dimensional flow, the $2\pi Y$ term is omitted

PRF(9) The net thrust coefficient, which is the CTG less (1) friction drag along the contour, and (2) subsonic losses. For a perfect gas and axisymmetric flow, the frictional drag coefficient is

$$DRAG = 1/2 \int \frac{C_f \, P \, M^2 \, \gamma}{P_o \cdot A^*} 2\pi Y \, dY$$

where: the coefficient of friction (C_f) at each point is determined from

$$C_f = C_{f_i} \left[1 + .72 \frac{\gamma - 1}{2} M^2 \right]^{-0.578}$$

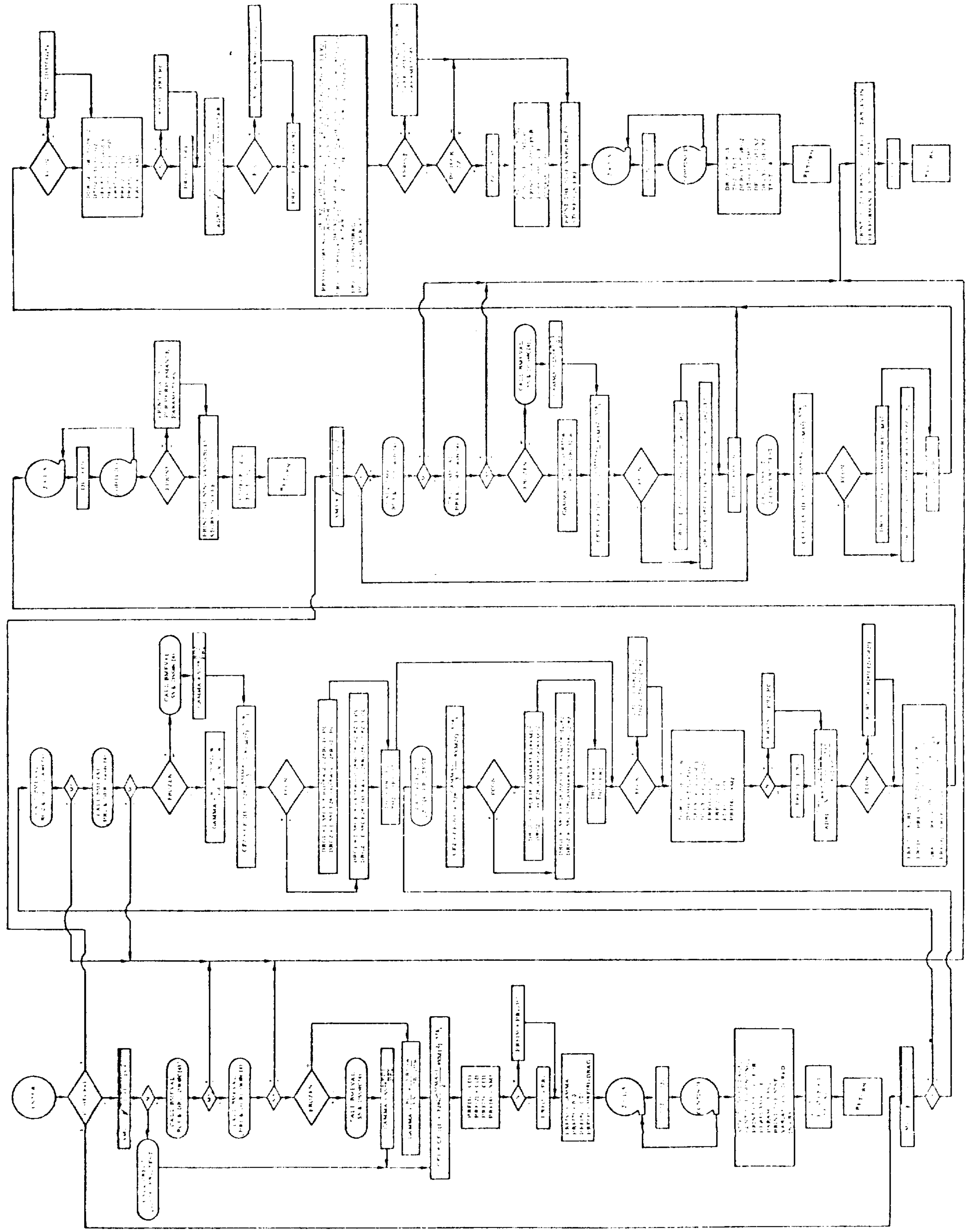
The equation for two-dimensional flow is the same except the $2\pi Y$ term is omitted. For an ideal gas

$$DRAG = 1/2 \int \frac{C_f \, W^2 \, V^2_{sonic} \, \rho}{P_o \cdot A^*} 2\pi Y \, dY$$

Again the $2\pi Y$ term is omitted for two-dimensional flow.

The subsonic thrust coefficient loss is an input parameter.

Subroutine PERFOR



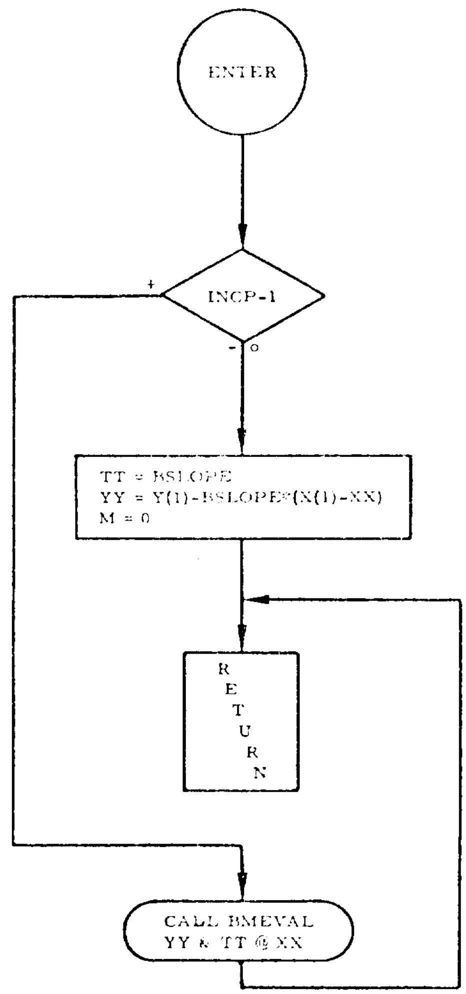
18. EVAL Subroutine

For a given value of X, the EVAL Subroutine either calculates the Y coordinate and slope of a conical nozzle contour, or calls the BMEVAL Subroutine if the contour has been beam fit. If the value in INCP $\leq$ 1, the following equations are used.

$$Y_i = Y_1 - (X_1 - X_i) Y'.$$

The value of Y' is the slope of the conical contour and must be stored in BSLOPE.

Subroutine EVAL



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19. BMFIT Subroutine

The BMFIT Subroutine is used to calculate sets of cubic coefficients for a spline curve fit through a series of input points. This method of curve fitting, commonly referred to as beam-fitting, is derived in Volume I. The calling sequence for the subroutine is:

CALL BMFIT (L,N,X,Y,EO,EN,A,B,C,D),

where:

L - A fixed point variable denoting one of the following moment or slope end-condition options

$L = 1$, $M_1 = EO$ and $M_N = EN$

$L = 2$, $M_1 = EO$ and $M_N = M_{N-1}$

$L = 3$, $M_1 = EO$ and $Y'_N = EN$

$L = 4$, $M_1 = M_2$ and $M_N = EN$

$L = 5$, $M_1 = M_2$ and $M_N = M_{N-1}$

$L = 6$, $M_1 = M_2$ and $Y'_N = EN$

$L = 7$, $Y'_1 = EO$ and $M_N = EN$

$L = 8$, $Y'_1 = EO$ and $M_N = M_{N-1}$

$L = 9$, $Y'_1 = EO$ and $Y'_N = EN$

N - A fixed point variable equal to the number of points to be fit

X - A single dimensional array containing the values of the independent variables

Y - A single dimensional array containing the values of the dependent variable

EO - The moment or slope at the leading end as required by the options $L = 1,2,3,7,8$, and 9. (EO is zero for $L = 4,5$ and 6.)

EN - The moment or slope at the trailing end as required by the options $L = 1,3,4,6,7$, and 9 (EN is zero for $L = 2,5$, and 8.)

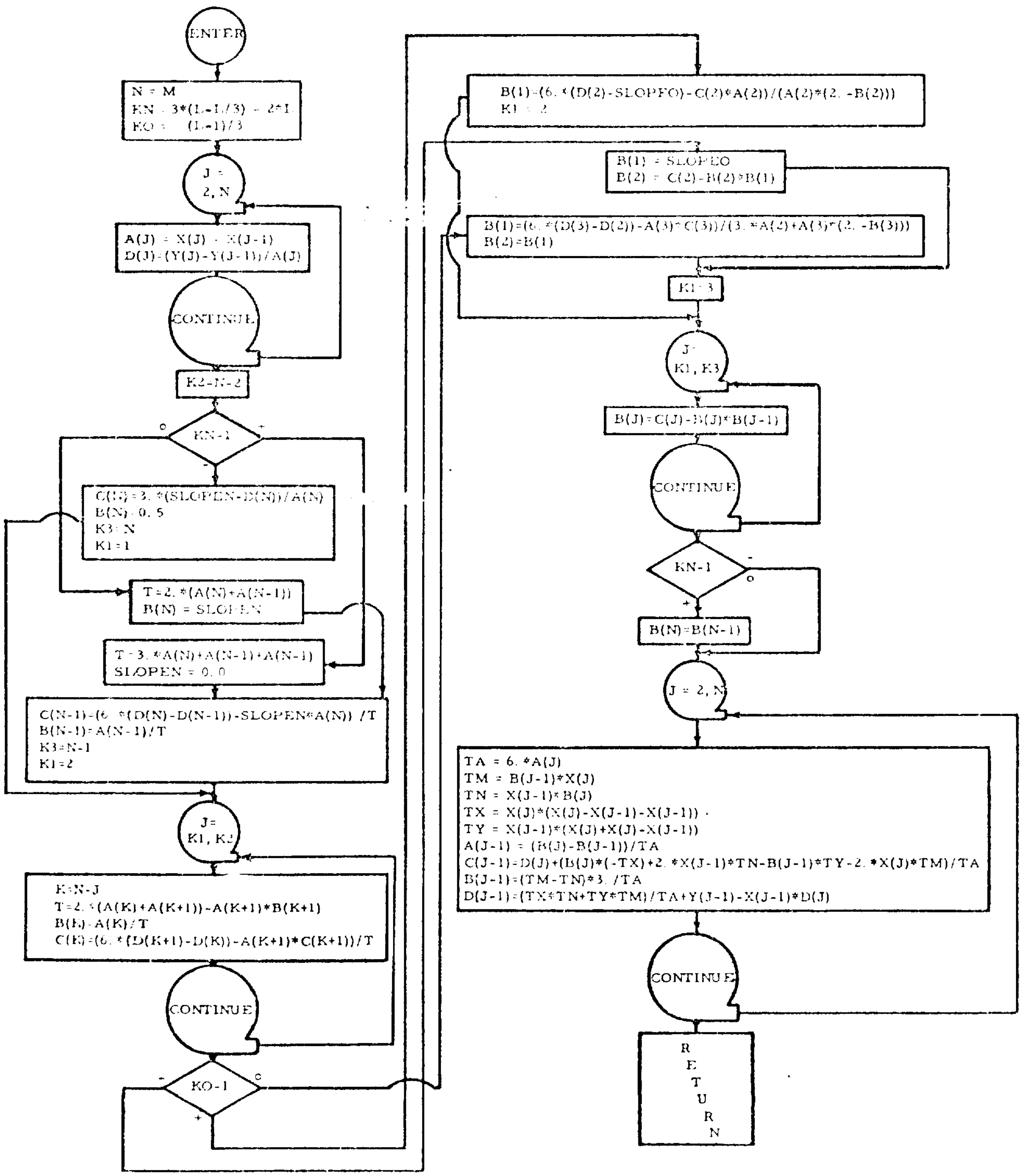
M_i - The moment at the i^{th} contour point

Y'_i - The slope at the i^{th} contour point.

A set of coefficients is calculated for the interval between each input point and then stored in the one dimensional arrays A, B, C, and D. For the i^{th} interval,

$$Y = A_i X^3 + B_i X^2 + C_i X + D_i .$$

Subroutine BMFIT



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20. BMEVAL Subroutine

After a curve that is represented by a set of input coordinates has been fit by the BMFIT Subroutine, the BMEVAL Subroutine is used to evaluate the curve and its first derivative for a given value of the independent variable. The subroutine solves the cubic equation $Y = AX^3 + BX^2 + CX + D$ after searching for the set of coefficients corresponding to the given value of the independent variable.

The eleven parameters that make up the call list of the BMEVAL Subroutine are as follows:

N - The number of coordinates describing the curve

X - Contains values of the independent variable in increasing order (maximum of 100)

Y - Corresponding values of the dependent variable (maximum of 100)

VALU - Value of the independent variable at which the curve is to be evaluated.

L - Error signal

L = -1, curve is out of range to the left

L = 0, curve is in range

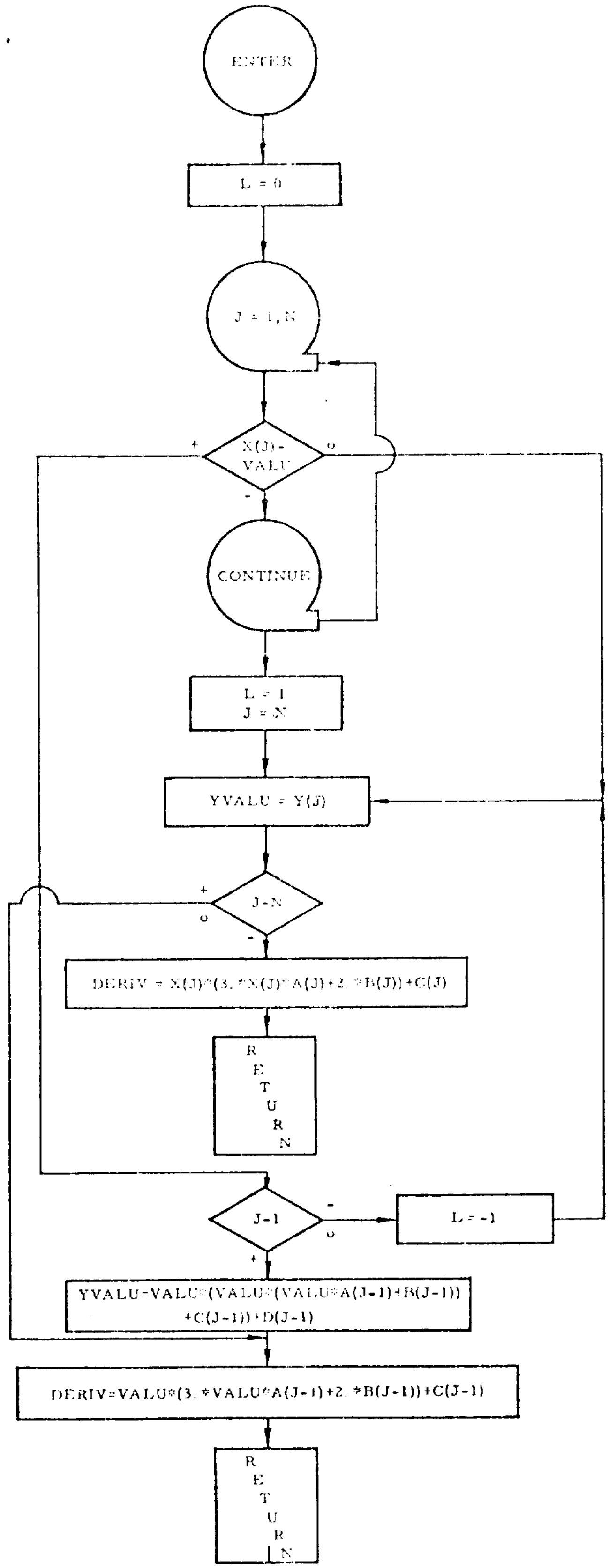
L = +1, curve is out of range to the right

A,B, C,D - Contain the coefficients of the cubic equation for intervals between each input coordinate (these coefficients are calculated in the BMFIT Subroutine)

YVALU - The calculated value of the dependent variable corresponding to VALU

DERIV - Calculated value of the first derivative corresponding to VALU.

Subroutine BMEVAL



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SECTION III
INPUT - OUTPUT

The INPUT Subroutine is used to initialize constants and to load the input data. After loading the required input, this subroutine reads any of the optional input with a scatter loading feature that terminates with an END card. The following is a detailed outline of the input procedure. A sample input sheet is shown in figure 16, which corresponds to the output results given in Paragraph B.

A. INPUT FORMAT

All data cards must contain the FORTRAN variable name in card columns 1-6 (adjusted to the left). For single-valued variables, the number must be in columns 7-16. For variables containing more than one value, the numbers are in field widths of ten columns, beginning in column 7 of each card (6 per card). Depending on the contour option and the gas model option, the parameters under Required Input must be input and in the order listed. The Optional Input has no particular order, and is terminated by a card with END in columns 1-3. Each Optional Input variable will equal the built-in value, unless another number is input. The built-in value is restored between multiple cases.

1. Required Input

a. General

TITLE — Any information to be printed as a heading at the begining of the output will be placed in columns 7-72.

PA — Ambient pressure ratio, P_a/Pc.

FBCUT — Cutoff value of X/R_t along free boundary

PWA 10171 REV. 2-62

GENERAL INPUT FORM

SHEET 1 OF

ANALYST SAMPLE INPUT ENGINEER EXT

JOB NO. COST CONTROL NO.

TITLE	SAMPLE TEST CASE					
PA	0.08					
FBCUT	2.0					
XNCP	17.0					
X	-.05912608	-.00927986	.03668003	.08409383	.1351359	.1908256
X	.2531009	.3240955	.4063814	.5033920	.6200231	.7632781
X	.9447142	1.184146	1.520901	1.677299	1.697410	
Y	.6085172	.5726654	.5408527	.5098878	.4786153	.4463421
Y	.4124487	.3765603	.3383042	.2970164	.2518671	.2028279
Y	.1485739	.08914635	.02943394	.02014031	.020	
RSLOPE	-.73415188					
ESLOPE	0.0					
GAMMA	1.4					
[illegible]E	2.0					
[illegible]M	1.07					
END						

Figure 16

b. Contour

XNCP — The number contour coordinates.

XNCP = 1.0, Conical nozzle

XNCP > 1.0, Contour is beam fit.

If XNCP = 1.0

X — The first X/R_t value on the contour

Y — The corresponding Y/R_t value

BSLOPE — The contour slope in degrees (must be negative).

If XNCP > 1.0

X — The X/R_t values in increasing order

Y — The corresponding Y/R_t values

BSLOPE — The contour slope at the first X/R_t

ESLOPE — The contour slope at the last X/R_t

c. Gas Model Option

(1) Perfect Gas (Constant Specific Heat Ratio)

GAMMA — Specific heat ratio

(2) Ideal Gas (Table of Gas Properties is Required and Must Follow the END Card)

(a) Local Sound Speeds Calculated by Program

XN — The number of cards in the table of gas properties

PC — Chamber pressure, psi

(b) Local Frozen Sound Speeds from Table Are Used

FROZEN — Must be non zero

XN — The number of cards in the table of gas properties

PC — Chamber pressure, psi

2. Optional Input

Name	Built-In Value	Description
TF	1.0	TF = 0.0, A starting Mach line to be loaded into BL(I,J) TF = 1.0, Program calculates a straight Mach line (for perfect gas only). TF = 2.0, Program calculates a Mach line of constant Mach number. The desired Mach number is in TM. If a starting line is to be loaded, it must follow the table of gas properties; or for a constant GAMMA it follows the END card
FCON	1.0	FCON = 1.0, Axisymmetric flow FCON = 0.0, Two-dimensional flow
XNUM	5.0	Number of points on starting Mach line
XCUT	0.0	Exit value of X/R_t along contour. If nothing is input, XCUT is set equal to last X value for beam fit contour or XCUT is the intersection point of contour and minimum Y/R_t (CYLHT) for conical nozzle.
FRRINT	0.0	FPRINT = 0.0, flow field is not printed FPRINT = 1.0, flow field is printed
CUTTOL	.0001	Tolerance on cutoff iteration
CF	.003	Incompressible coefficient of skin friction along contour
DRAG	.011	Thrust loss at the throat
DELTW	.01	Expansion increment of velocity. If nothing is input, DELTW = .01 for a perfect gas DELTW = .005 for an ideal gas
CYLHT	.01	Minimum value of Y/R_t along contour
DELX	.1	Maximum distance between adjacent Mach lines along contour

Name	Built-In Value	Description
WTOL	.000005	Iteration tolerance on Mach line intersection
TM	1.005	Mach number for a constant Mach number starting line
TOL	.0000001	Minimum Y/R_t distance between starting line and expansion point
DELTX	.001	X/R_t increment to shift starting Mach line
PHI	-.5	Angle of rotation for interior intersections, radians

A card with END in columns 1-3 must follow the last optional input card.

For an ideal gas, a table of gas properties must be input. The first card is a title card describing the gas model, and the second card contains in columns 2-15 the specific impulse, $\left(\frac{lb_{f-sec}}{lbm}\right)$, at the throat. Each of the remaining cards must contain corresponding values of specific impulse $\left(\frac{lb_{f-sec}}{lbm}\right)$, pressure (psi), and density (lbm/ft^3) in columns 2-15, 16-29, and 30-43, respectively, with the local sound speed (ft/sec) in columns 58-71.

If a starting down Mach line is to be input, the values of X, Y, tan θ, W, and tan α at each point on the line must be on a separate card with the numbers in field widths of ten beginning in column 7. The first six columns of each card is used to identify the FORTRAN variable (BL).

B. OUTPUT DESCRIPTION

The first page of the output includes the title, values of important input parameters, an evaluation of the nozzle contour if any portion has been beam fit, and the values of X, Y, tan θ, W, and tan α at points along the starting down Mach line. If there is no flow field printout, the following lines of output are the nine performance parameters at points of Mach line intersections with the contour, as explained in the PERFOR Subroutine, and points along the free boundary. A sample of the program output is shown in figures 17a, 17b, and 17c.

C. PROCEDURES FOR CORRECTING PROGRAM FAILURES

After thoroughly checking the input when a program failure occurs, the beam fit contour should be carefully inspected. Because the beam fit procedure forces a continuous curve to pass exactly through each input point, a small error in any of the X and Y values can cause waves in the contour. It is often helpful to make an enlarged plot of the contour to smooth the curve and read the input coordinates more accurately.

Listed below are several types of program failures with explanations and corrective procedures.

1. Crossing of Like Mach Lines

If the contour curve is an accurate representation, the crossing of like Mach lines is normally legitimate and indicates a strong shock in the flow field. A weak shock can occur and cause like Mach lines to cross, but the program will not fail. In this case, the Mach lines will fold back and the remaining results are still comparatively accurate. Another case of crossing Mach lines could be an incorrect starting Mach line.

2. Negative tan α

This failure is caused by an attempt to calculate tan α from a subsonic velocity. Usually, this happens only when a strong shock occurs.

3. Iteration Failure in the Interior Subroutines

If after 100 iterations, the solution of a Mach line intersection has not converged, the program will fail the case. If the convergence tolerance is not too small, this failure is normally the result of either too large a mesh size or a bad choice of rotation angle. The mesh size is made smaller by increasing the number of points on the starting Mach line, decreasing the maximum distance between adjacent Mach lines along the contour, or decreasing the expansion increment. If the coordinate system is rotated to avoid the position of a Mach line, the slope of the opposite Mach line can be rotated into an undesirable position. A better choice of rotation angle can be input for PHI.

4. Iteration Failure in BOUNDD Subroutine

This is due to a discontinuity or poor curve fit of the nozzle contour. This iteration will also fail if the mesh size is too large.

SINGLE EXPANSION PLUG NOZZLE PERFORMANCE

CONSTANT GAMMA,
AXISYMMETRIC FLOW

SAMPLE TEST CASE

* * * I N P U T * * *

SPECIFIC HEAT RATIO (GAMMA).=	1.40000	NO. OF PTS. ON INITIAL MACH LINE (XNUM).=	5.00000
AMBIENT PRESSURE RATIO (PA).=	0.08000	VELOCITY EXPANSION INCREMENT (DELTW)....=	0.00500
FLOW CONDITION (FCON).......=	1.00000	FLOW FIELD PRINT OPTION (FPRINT)........=	0.
FRICTION COEFFICIENT (CF)...=	0.00300	X/R FOR LAST DESIRED POINT (XCUT).......=	1.69741
CHAMBER LOSSES (DRAG).......=	0.01100	MAXIMUM X/R INCREMENT (DELX)............=	0.10000
FREE BOUNDARY CUTOFF (FBCUT)=	2.00000	FROZEN SOUND SPEED OPTION (FROZEN)......=	0.

* * * CONSTANT THROAT VALUES * * *

MFLOW = 0.15305 SONICV= 0.40825 RHOSON= 0.63594 ASTAR = 0.59137 WN = 0.71697

* * * PLUG CONTOUR * * *

X	Y	EVALUATED Y	DY/DX
-0.05912608	0.60851720	0.60851720	-0.73415187
-0.00927985	0.57266539	0.57266539	-0.70618470
0.03668003	0.54085270	0.54085270	-0.67453981
0.08409382	0.50988780	0.50988780	-0.63215115
0.13513590	0.47861530	0.47861530	-0.59570146
0.19082560	0.44634210	0.44634210	-0.56295978
0.25310089	0.41244870	0.41244870	-0.52577011
0.32409550	0.37656029	0.37656029	-0.48579542
0.40638140	0.33830420	0.33830420	-0.44537337
0.50339200	0.29701640	0.29701640	-0.40752262
0.62002309	0.25186709	0.25186709	-0.36598878
0.76327810	0.20282789	0.20282789	-0.32180592
0.94471420	0.14857390	0.14857390	-0.27545428
1.18414599	0.08914635	0.08914635	-0.22584755
1.52090099	0.02943394	0.02943394	-0.10481025
1.67729899	0.02014031	0.02014031	-0.01345985
1.69740999	0.02000000	0.02000000	-0.00000131

* * * THROAT MACH LINE OR FIRST DOWN MACH LINE * * *

X	Y	T	W	A
-0.	0.10000005E 01	-0.41514544E-00	0.43164465E-00	0.26270349E 01
-0.15071024E-01	0.86655325E 00	-0.49612191E-00	0.43164465E-00	0.26270349E 01
-0.30142049E-01	0.74924373E 00	-0.57966133E 00	0.43164465E-00	0.26270349E 01
-0.45213073E-01	0.67024614E 00	-0.65659023E 00	0.43164465E-00	0.26270349E 01
-0.60284097E-01	0.60936736E 00	-0.73415188E 00	0.43164465E-00	0.26270349E 01

Figure 17a

PERFORMANCE

X/R	Y/R	TAN THETA	MACH NO.	P/PC	GAMMA	AS/A*	CTG	CTN
-0.0602841	0.6093674	-0.7341519	1.0700000	0.4859517	1.4000000	0.	1.2047479	1.1937479
-0.0290908	0.5867497	-0.7159779	1.0744347	0.4833256	1.4000000	0.0779325	1.2269199	1.2158524
-0.0162003	0.5775636	-0.7094075	1.0889674	0.4748203	1.4000000	0.1090970	1.2355915	1.2244967
-0.0032276	0.5684000	-0.7032693	1.1037184	0.4661976	1.4000000	0.1398751	1.2439470	1.2328249
0.0096239	0.5594079	-0.6958183	1.1174886	0.4582595	1.4000000	0.1697883	1.2518731	1.2407243
0.0220792	0.5507948	-0.6869620	1.1290018	0.4516821	1.4000000	0.1982176	1.2592286	1.2480543
0.0364075	0.5410365	-0.6747831	1.1384253	0.4463398	1.4000000	0.2302242	1.2673150	1.2561115
0.0502792	0.5317647	-0.6620524	1.1470468	0.4414849	1.4000000	0.2604926	1.2747814	1.2635502
0.0645448	0.5224119	-0.6492418	1.1568766	0.4359885	1.4000000	0.2909008	1.2820972	1.2708380
0.0791891	0.5129987	-0.6363886	1.1676218	0.4300281	1.4000000	0.3213814	1.2892346	1.2779469
0.0940111	0.5036600	-0.6238993	1.1804088	0.4230014	1.4000000	0.3514988	1.2960839	1.2847679
0.1095477	0.4940601	-0.6121061	1.1944637	0.4153619	1.4000000	0.3823115	1.3028739	1.2915286
0.1254961	0.4843849	-0.6014406	1.2109957	0.4064901	1.4000000	0.4131750	1.3094538	1.2980790
0.1419866	0.4745477	-0.5917952	1.2295551	0.3966797	1.4000000	0.4443115	1.3158611	1.3044563
0.1590737	0.4645198	-0.5819036	1.2479198	0.3871297	1.4000000	0.4757729	1.3221014	1.3106661
0.1768260	0.4542826	-0.5714025	1.2656265	0.3780718	1.4000000	0.5076119	1.3281862	1.3167198
0.1952647	0.4438490	-0.5602636	1.2826033	0.3695271	1.4000000	0.5397880	1.3341087	1.3226107
0.2144396	0.4332171	-0.5486971	1.2992242	0.3612949	1.4000000	0.5723057	1.3398702	1.3283401
0.2344105	0.4223783	-0.5367882	1.3156435	0.3532930	1.4000000	0.6051810	1.3454729	1.3339102
0.2552400	0.4113254	-0.5245176	1.3318431	0.3455257	1.4000000	0.6384243	1.3509172	1.3393213
0.2769981	0.4000501	-0.5119474	1.3479365	0.3379352	1.4000000	0.6720476	1.3562036	1.3445741
0.2997616	0.3885430	-0.4991268	1.3640152	0.3304772	1.4000000	0.7060609	1.3613319	1.3496681
0.3236140	0.3767943	-0.4860555	1.3800943	0.3231446	1.4000000	0.7404719	1.3663009	1.3546025
0.3486483	0.3647925	-0.4728956	1.3964518	0.3158139	1.4000000	0.7752868	1.3711093	1.3593756
0.3749705	0.3525182	-0.4598583	1.4134371	0.3083392	1.4000000	0.8105156	1.3757559	1.3639865
0.4026998	0.3399469	-0.4470107	1.4311595	0.3006889	1.4000000	0.8461664	1.3802388	1.3684332
0.4319699	0.3270496	-0.4344025	1.4496973	0.2928489	1.4000000	0.8822426	1.3845562	1.3727138
0.4629297	0.3137941	-0.4220742	1.4691077	0.2848165	1.4000000	0.9187385	1.3887055	1.3768260
0.4957499	0.3001407	-0.4101355	1.4895558	0.2765492	1.4000000	0.9556419	1.3926845	1.3807675
0.5306214	0.2860461	-0.3982085	1.5103879	0.2683303	1.4000000	0.9929247	1.3964908	1.3845360
0.5677369	0.2715076	-0.3851377	1.5297409	0.2608775	1.4000000	1.0305070	1.4001170	1.3881241
0.6072979	0.2565546	-0.3707282	1.5474512	0.2542100	1.4000000	1.0682723	1.4035550	1.3915239
0.6495345	0.2412253	-0.3553398	1.5643456	0.2479843	1.4000000	1.1060939	1.4067946	1.3947254
0.6947481	0.2254999	-0.3405819	1.5832910	0.2411576	1.4000000	1.1438746	1.4098310	1.3977237
0.7433319	0.2092961	-0.3268242	1.6048989	0.2335690	1.4000000	1.1815297	1.4126600	1.4005150
0.7957593	0.1925039	-0.3138565	1.6286734	0.2254593	1.4000000	1.2189337	1.4152795	1.4030972
0.8525383	0.1750861	-0.2995953	1.6510347	0.2180571	1.4000000	1.2558506	1.4176800	1.4054611
0.9142188	0.1570982	-0.2835766	1.6716239	0.2114316	1.4000000	1.2919412	1.4198494	1.4075951
0.9814611	0.1386289	-0.2659322	1.6914191	0.2052307	1.4000000	1.3268127	1.4217736	1.4094851
1.0551699	0.1196715	-0.2489187	1.7149653	0.1980667	1.4000000	1.3600554	1.4234447	1.4111239
1.1366134	0.1000554	-0.2333581	1.7432464	0.1897603	1.4000000	1.3911818	1.4248597	1.4125089
1.2275147	0.0795060	-0.2183302	1.7726169	0.1814698	1.4000000	1.4194793	1.4260185	1.4136409
1.2774760	0.0688750	-0.2067126	1.7844620	0.1782210	1.4000000	1.4322959	1.4264981	1.4141085
1.3305512	0.0583074	-0.1909023	1.7924708	0.1760547	1.4000000	1.4439346	1.4269005	1.4145000
1.3868538	0.0481224	-0.1702245	1.7976980	0.1746539	1.4000000	1.4542319	1.4272218	1.4148117
1.4466268	0.0387126	-0.1438716	1.8060011	0.1724500	1.4000000	1.4631170	1.4274618	1.4150434
1.5100858	0.0306007	-0.1109334	1.8268913	0.1670182	1.4000000	1.4706154	1.4276237	1.4151983
1.5771635	0.0244482	-0.0723465	1.8754368	0.1550047	1.4000000	1.4768857	1.4277164	1.4152853
1.6470965	0.0208176	-0.0313692	1.9597404	0.1360542	1.4000000	1.4822459	1.4277572	1.4153215

EXPANSION BEYOND CUT OFF FOR UNDER-EXPANDED CONDITION

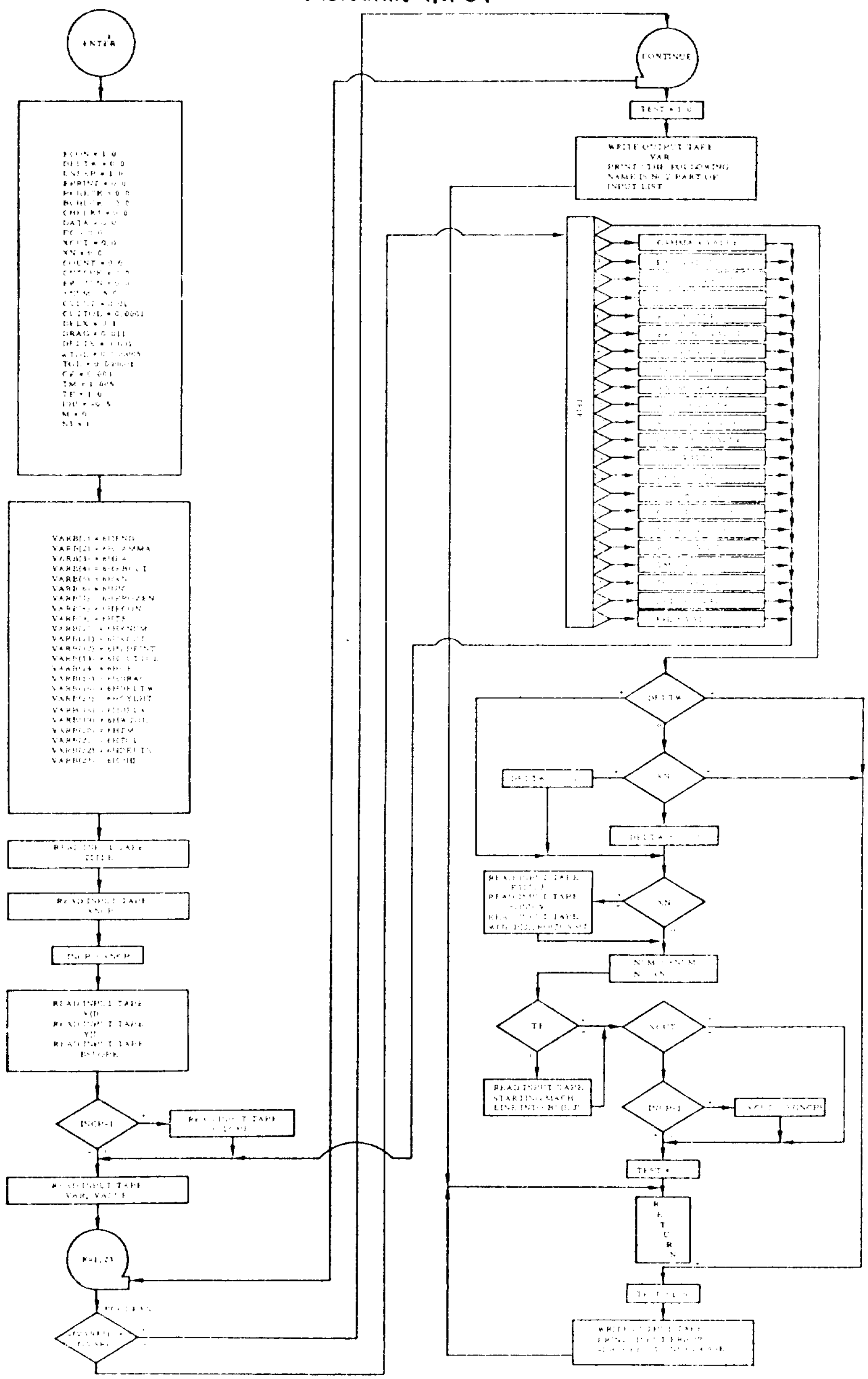
Figure 17b

FREE BOUNDARY POINTS

X	Y	T	W	A	
-0.	0.10000005E 01	0.19319353E-00	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.24656407E-00	0.10426559E 01	0.15002692E-00	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.48806898E-00	0.10751494E 01	0.12131292E-00	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.65441841E 00	0.10940201E 01	0.10555486E-00	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.78272272E 00	0.11069923E 01	0.96651554E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.86405717E 00	0.11149576E 01	0.99215480E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.90824834E 00	0.11192848E 01	0.96623693E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.95271339E 00	0.11235144E 01	0.93619571E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.99315428E 00	0.11272585E 01	0.91555652E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.10277291E 01	0.11304121E 01	0.90877470E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.10606601E 01	0.11334260E 01	0.92165530E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.10900224E 01	0.11361594E 01	0.94017625E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.11207502E 01	0.11390708E 01	0.95480438E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.11525647E 01	0.11421264E 01	0.96609844E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.11865023E 01	0.11454098E 01	0.96891864E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.12238004E 01	0.11490183E 01	0.96300964E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.12655751E 01	0.11529936E 01	0.94307626E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.13114624E 01	0.11572476E 01	0.91101957E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.13576086E 01	0.11613809E 01	0.88036418E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.14031099E 01	0.11653277E 01	0.85450391E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.14478233E 01	0.11691023E 01	0.83380212E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.14924986E 01	0.11727871E 01	0.81595123E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.15375037E 01	0.11764231E 01	0.79998723E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.15828695E 01	0.11800202E 01	0.78594568E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.16289102E 01	0.11836088E 01	0.77307680E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.16759124E 01	0.11872133E 01	0.76075618E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.17240081E 01	0.11908432E 01	0.74882063E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.17739025E 01	0.11945458E 01	0.73552473E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.18264883E 01	0.11983692E 01	0.71873548E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.18821769E 01	0.12023135E 01	0.69781867E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.19413356E 01	0.12063664E 01	0.67237487E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01
0.20043329E 01	0.12105051E 01	0.64216287E-01	0.71696845E 00	0.48286125E-00	0.22997809E 01

Figure 17c

Subroutine INPUT



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APPENDIX A
SYMBOL TABLE

A^*	-	Theoretical throat area
c	-	Local speed of sound
Cf	-	Coefficient of friction
C_{TG}	-	Gross thrust coefficient
C_{TN}	-	Net thrust coefficient
I_s	-	Specific impulse
M	-	Mach number
P	-	Pressure
V_{max}	-	Maximum velocity
V_{sonic}	-	Sonic velocity at the throat
W	-	Velocity ratio; either V/V_{max} or V/V_{sonic}
$\dot{\omega}$	-	Mass flow rate
α	-	Mach angle
γ	-	Ratio of specific heats
ρ	-	Density
σ	-	= 1 for axisymmetric flow = 0 for two dimensional flow
θ	-	Angle between velocity vector and axis of symmetry

APPENDIX B
FORTRAN SYMBOL TABLE
(COMMON)

Variable	Dimension	Description
A1	100	Coefficients of X^3 from contour beam fit
A2	100	Coefficients of X^3 from ρ vs W beam fit
A3	100	Coefficients of X^3 from P vs W beam fit
A4	100	Coefficients of X^3 from c vs W beam fit
AL	200,5	Variable used to store X, Y, tan θ, W, and tan α on a down Mach line
A	5	For storing X, Y, tan θ, W, and tan α at a point on an up Mach line
ASTAR	-	Theoretical throat area
B1	100	Coefficients of X^2 from contour beam fit
B2	100	Coefficients of X^2 from ρ vs W beam fit
B3	100	Coefficints of X^2 from P vs W beam fit
B4	100	Coefficients of X^2 from c vs W beam fit
BL	200,5	Variable used to store X, Y, tan θ, W, and tan α on a down Mach line
B	5	For storing X, Y, tan θ, W, and tan α at a point on a down Mach line
BSLOPE	-	The initial slope of the contour
C1	100	Coefficients of X from contour beam fit
C2	100	Coefficients of X from ρ vs W beam fit
C3	100	Coefficients of X from P vs W beam fit
C4	100	Coefficients of X from c vs W beam fit
CF	-	Incompressible coefficient of skin friction
CHECK3	-	Indicates the final contour point has been calculated in Section III of the flow field

Variable	Dimension	Description
CL	200,6	Variable used to store X, Y, tan θ, W, and tan α along the free boundary
C	5	For storing X, Y, tan θ, W, and tan α usually at a point of Mach line intersection
CTG	-	Gross thrust coefficient at the first contour point
CUTCHK	-	Signal inside the BOUNDD Subroutine to indicate input contour has been exceeded
CUTTOL	-	Tolerance for the cutoff point iteration
CYLHT	-	Minimum Y/R_t value for the contour
D1	100	Contains constants from contour beam fit
D2	100	Contains constants from ρ vs W beam fit
D3	100	Contains constants from P vs W beam fit
D4	100	Contains constants from c vs W beam fit
DELTW	-	Expansion increment of velocity ratio
DELTX	-	Shifting increment of ΔX for starting Mach line iteration
DELX	-	Maximum distance between Mach lines along contour
DL	200,5	The last down Mach line in the expansion region
DRAG	-	Accumulated drag along the contour
D	5	For storing X, Y, tan θ, W, and turn α at previous points along the contour
ESLOPE	-	The end slope of the contour beam fit
FBCUT	-	Cutoff point along the free boundary
FCON	-	Indicates either two-dimensional or axisymmetric flow
FPRINT	-	Indicates whether flow field is to be printed

Variable	Dimension	Description
FROZEN	-	For the ideal gas option, this indicates whether local frozen sound speed is used
GAMMA	-	Specific heat ratio when constant
INCP	-	Fixed point value of the number of points on the contour
J1	-	Number of the first supersonic velocity in the table of gas properties
NCP	-	Number of points input for the contour
NFBP	-	The number of free boundary points
N	-	Number of cards making up the table of gas properties
NUM	-	The number of points on the starting down Mach line
PA	-	Ratio of ambient pressure to chamber pressure
PC	-	Chamber pressure, psi
P	100	Contains the values of pressure in table of gas preperties
RHOSON	-	Density evaluated at the sonic velocity
RO	100	Contains the values of density in table of gas properties
SONICV	-	Sonic velocity
TF	-	Indicates the type of starting Mach line
TITLE	10	To store and print title card
TM	-	Mach number for constant Mach number starting line.
UNEXP	-	Indicates that nozzle flow is under expanded
VS	100	Contains the values of local frozen sound speed in table of gas properties
WN	-	Minimum velocity ratio corresponding to PA

Variable	Dimension	Description
W	100	Contains the values of velocity ratio in table of gas properties
WSOC2	-	Sonic velocity squared
WTOL	-	Iteration tolerance on interior intersections
XNUM	-	Floating point value of NUM
X	100	Contains X coordinates of input contour
Y	100	Contains Y coordiantes of input contour